PROCESSING OF UNDERSAMPLED BLADE SIGNALS

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Abstract: *The paper deals with a method of processing samples of instantaneous positions of tips of blades of a turbine wheel when passing along a sensor attached to the stator of the turbine. The signals are highly undersampled, since the natural frequencies of blades are much higher than the sampling frequency. The paper reveals the way of signal reconstruction based on the a priory knowledge of the blade natural frequencies.*

Key words: *turbine blades, signal processing, fatigue, residual life*

1. Introduction

Much effort has been devoted to ensure a total reliability of nuclear power plants. The reliability of of the plant requires the reliability of its single parts as well. This is the reason, why the attention has been paid also for a vibration of individual parts of turbosets, blades included. For the purpose, special monitoring systems have been designed. The new systems are based on the exact time measurements (see [1, 2]). The clock pulses of the high frequency (10-100 MHz) are counted and the content of the counter is recorded in a moment, when a blade tip is passing the sensor fixed to the stator. The resulting sequence of times should be processed into deflections from the equilibrium position, and those into stresses in critical points of a structure. In case the dynamic stresses overcome certain level, the danger of fatigue damage of blading becomes real. The monitoring system should give a message on it to the control room of the machine.

2. Fourier Analysis of Sampled Signals

Sampling the signal $h(t)$ of blade tip deflections by a sampling frequency $f_s = 1/T$, where T is the sampling period, corresponds to the multiplication of $h(t)$ by the Dirac comb

$$\delta_T(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT), \tag{1}$$

which is a periodic series of Dirac pulses shifted mutually by T in time. The operation generates a time series

$$h_T(t) = h(t)\,\delta_T(t) = \sum_{k=-\infty}^{\infty} h(kT)\,\delta(t - kT), \tag{2}$$

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Since the Fourier transform (FT) of the Dirac comb

$$\mathsf{F}\{\, h_T(t) = \frac{1}{T}\,\delta_{\frac{1}{T}}(f)\,\} \tag{3}$$

is a (scaled) Dicac comb in the frequency domain with the repetition period $f_s = 1/T$, the Fourier transform of the sampled signal is

$$\begin{aligned}\mathsf{F}\{\, h_T(t)\,\} &= \int_{-\infty}^{\infty} h(t)\,\delta_T(t)\,e^{-i2\pi f t}\,dt = \int_{-\infty}^{\infty} h(t)\sum_{n=-\infty}^{\infty}\delta(t-nT)\,e^{-i2\pi f t}\,dt \\ &= \frac{1}{T}\,H(f) * \delta_{\frac{1}{T}}(f) = H_{\frac{1}{T}}(f)\end{aligned} \tag{4}$$

It is a convolution of the Fourier transform of the original function $h(t)$ with the Dirac comb in the frequency domain. It is seen from the equation (3) that the Fourier transform of the sampled signal is an infinite sum of all copies of Fourier transforms shifted by multiples of sampling frequency f_s. In order to avoid the interference of the side frequency bands, the sampling frequency f_s should be chosen at least as a double the highest frequency f_h in the signal,

$$f_s \geq 2\,f_h \tag{5}$$

The formula (5) expresses the well known Shannon sampling theorem.

The convolution brings very serious problems, if the signal is undersampled, what means, if its highest frequency contained in the signal is higher than half the sampling frequency $f_s = 1/T$. The frequency components are then shifted to another frequencies, what is called ,,aliasing" or ,,folding". The phenomenon is well demonstrated in the figure.

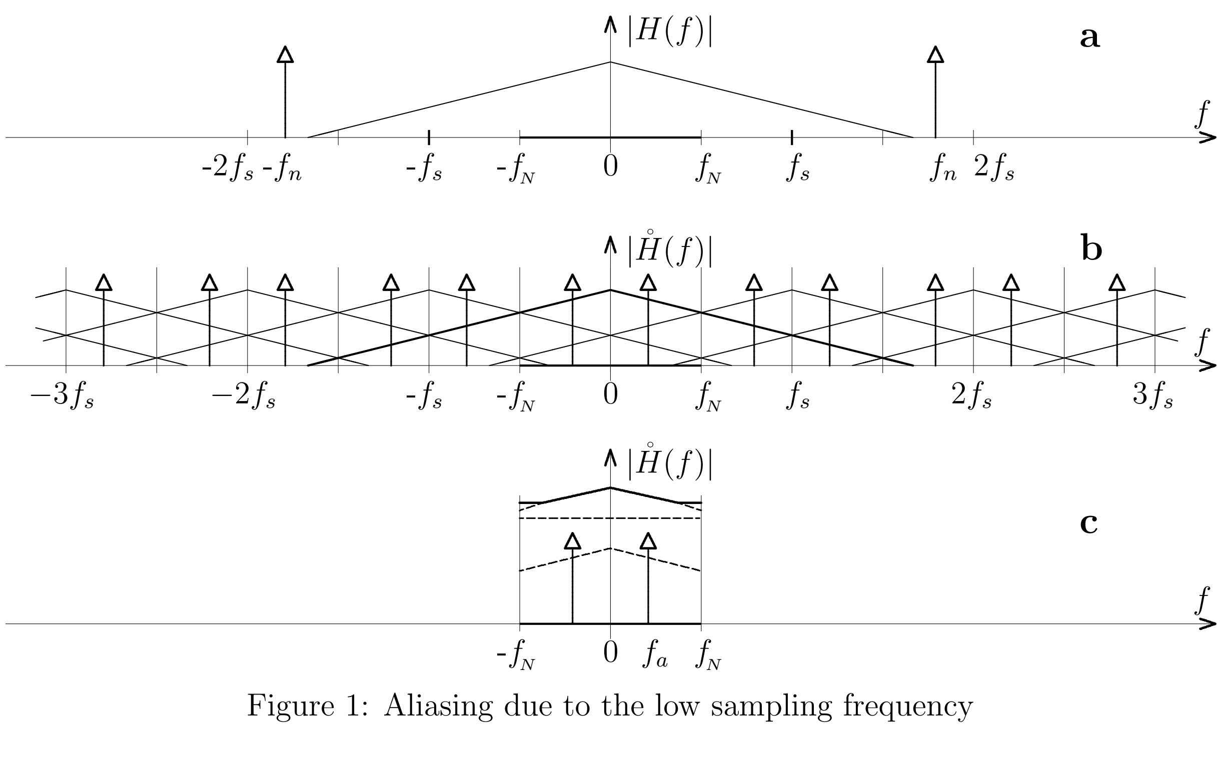

Figure 1: Aliasing due to the low sampling frequency

The low sampling frequency f_s caused a repetition of the original FT (in Fig.1a) into side bands (Fig. 1b) and a summing contributions of all bands into the basic frequency band (-f_N, f_N)) (in Fig 1c), where $f_N = f_s/2$ is so called Nyquist frequency.

The individual frequency components f_n are moved to another ones, aliased frequencies f_a, which may be evaluated as

$$f_a = f_n - f_s \,\mathrm{round}\left(\frac{f_n}{f_s}\right) \tag{6}$$

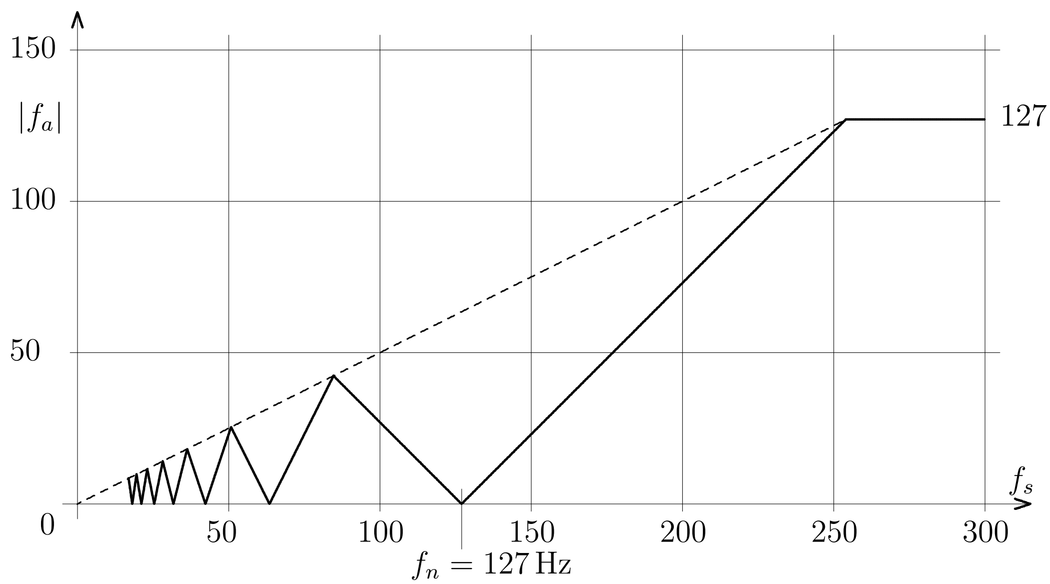

Figure 2: Aliased frequency f_a as a function of a sampling frequency f_s

The figure shows as an example how the sampling frequency influences the aliased frequency under the fixed actual frequency $f_n = 127\,\mathrm{Hz}$.

3. Reconstruction of the Vibrating Blade Signals

It is well known that a signal may not be reconstructed until the sampling frequency does not fulfill the sampling theorem. This fact would disable any attempts to estimate a fatigue life of the rotating blades, the signals of which are expressively undersampled, because of no knowledge of the real signal, the peaks of which are determining a speed of fatigue damage cumulation. Fortunately, there is a way at hand, how to support an estimating of the residual life of blades.

The system under observation, a turbine wheel with blades, is well known from the preoperation investigations during iys development. This means that the resonance frequencies $\hat{f}_n$ of rotating blades are known in forward from the numerical and experimental analyses. The rotational speed and the number of sensors in the stator for measuring blades determine the sampling frequency. Hence, the estimates of aliased frequencies $\hat{f}_a$ corresponding the resonant frequencies of blades in the Fourier transform of slowly sampled signals will be

$$\hat{f}_a = \hat{f}_n - f_s \,\mathrm{round}\left(\frac{\hat{f}_n}{f_s}\right) \tag{7}$$

Looking through the Fourier transform, one may find peaks of its modules on the frequencies f_a close to the $\hat{f}_a$. Those may be recalculated using formula (6) onto the frequencies f_n, which are the real resonance frequencies during the observation. Taking the peak value and next n_b frequency components close to it from both sides, and transferring them into a new Fourier spectrum $H_{\frac{k}{T}(f)}$ of the appropriate frequency width kf_s, one get a reconstructed Fourier transform of the original signal as if it were sampled by by a k multiple of the real sampling frequency f_s. If the contents of the aliased FT after picking up the resonance bands is not virtually empty, one may decide, how to spread the rest of spectra onto the frequency interval of the reconstructed Fourier transform $H_{kf_s}(f)$. The reconstructed signal, which aproximates the original one of the blade, is obtained by the inverse Fourier transform of the reconstructed Fourier spectrum $H_{kf_s}(f)$:

$$h_{T/k} = \mathsf{F}^{-1}\{\, H_{kf_s}(f) \,\} \tag{8}$$

The signal may be used for any further analysis of peaks and a damage of the material of the blades.

4. Measurements and Data Processing

The above mentioned method of a vibrating blade signal reconstruction has been used for processing both simulated and real measurements. Every measurement consists of a series of clock counts, say, 100 samples per every blade, complemented by 100 clock counts of the fixed marker (datum) attached to the rotor.

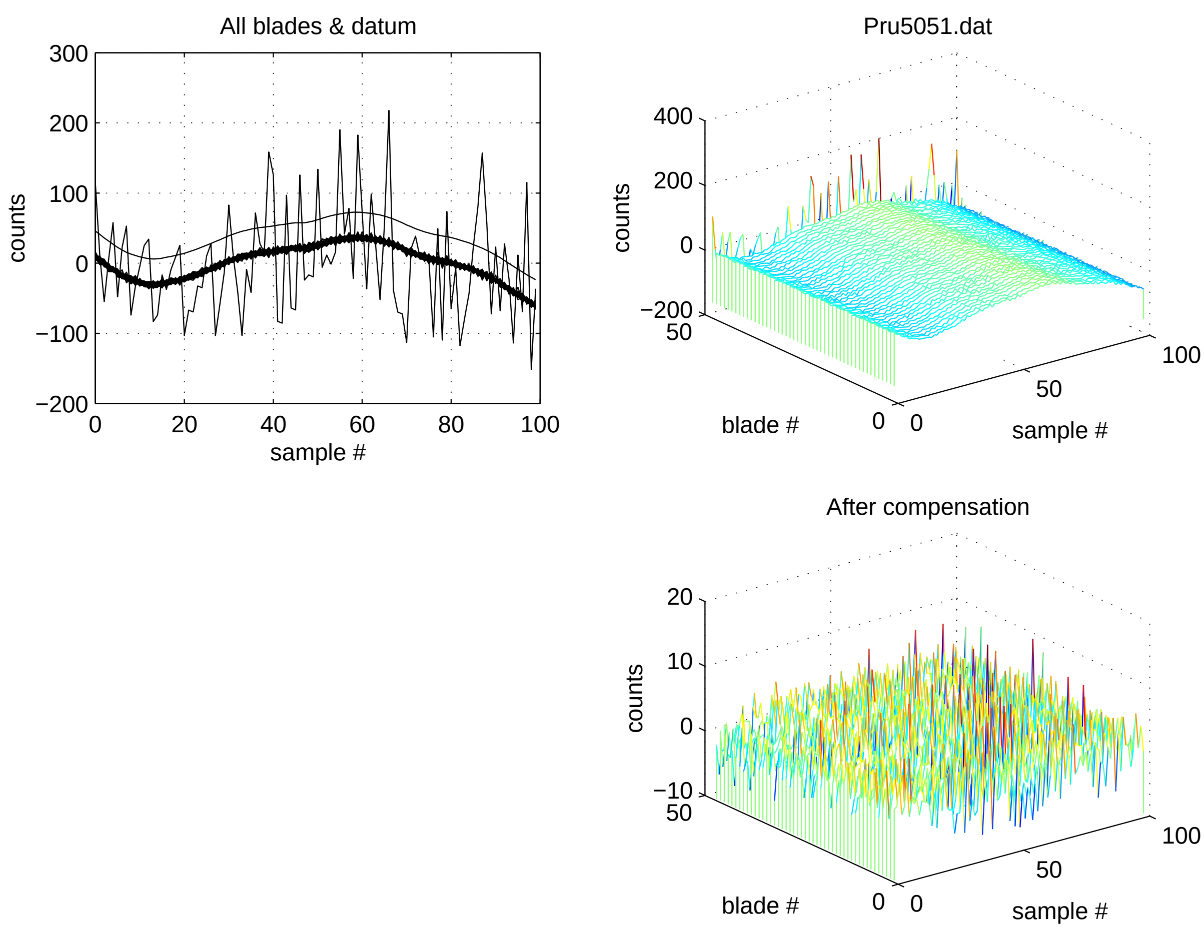

Figure 3: Measured signals of vibrating blades and datum

Should the rotational speed were constant, and the blades were stiff, the counts would create a sequence of numbers with a constant step, a linear function. After removing the linear trend from the real data, the resulting series will carry an information on time differences from the equilibrium, which correspond the attitudes of the blade tip from the equilibrium. The measurement taken under real conditions brings another problem associated with a nonstationarity of the speed during the measurement. This phenomena is seen on the Figure 3a. The thick line is the collection of signals taken from all blades. the averaged signal out of them is drawn by the thin line shifted by 50 counts higher. The line with high peaks correspond to the datum signal. The 3-D plot of one measurement is given in Fig. 3b. Fig 3c gives the view of efective signals of all blades after removing the nonstationarity.

As soon as the nonstationarity is removed, the arbitrary blade signal may be processed in the above mentioned way.

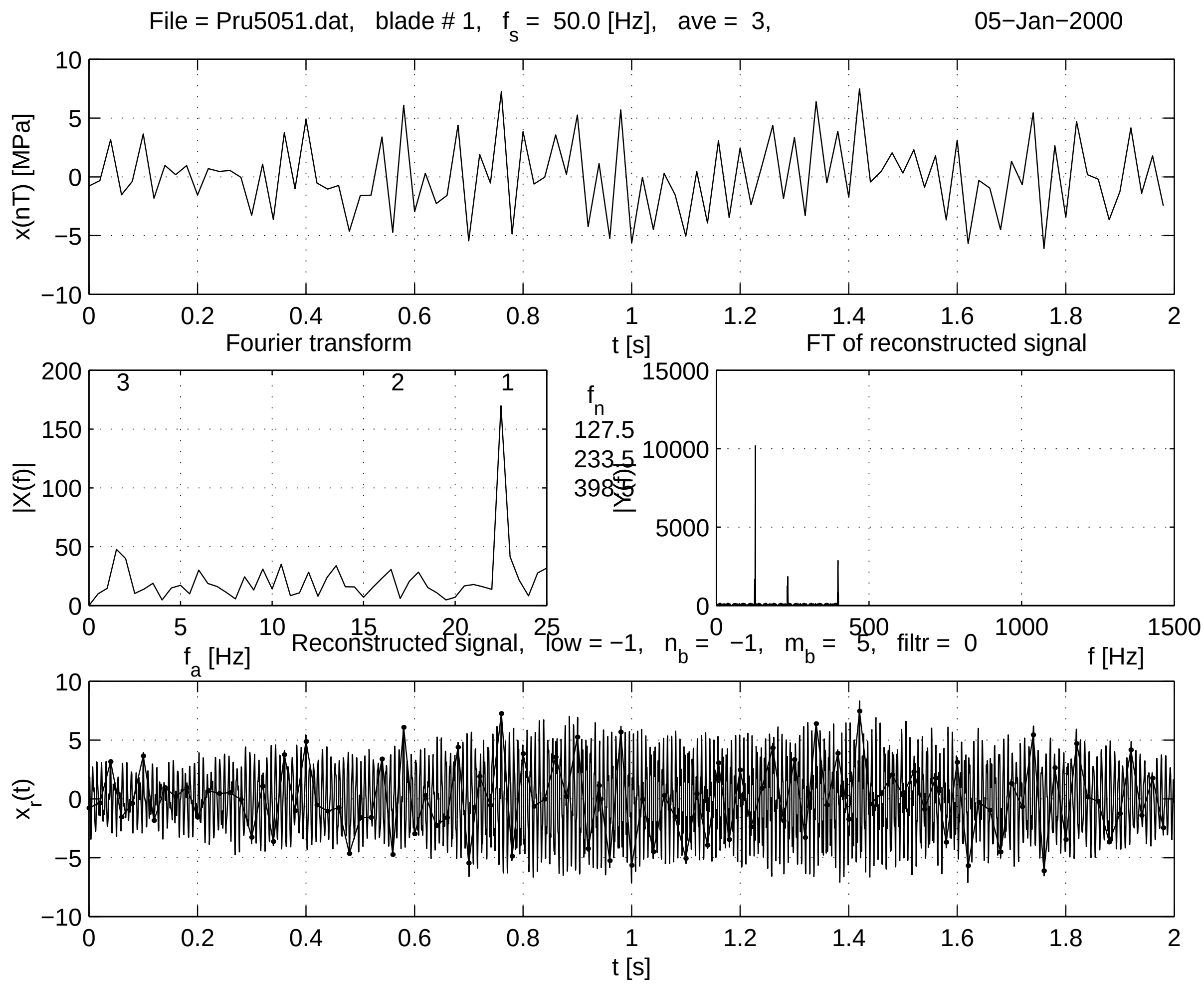

Figure 4: Slowly sampled signal, its FT, reconstructed FT, and reconstructed signal

5. Conclusions

The method presented in this contribution proved to be useful for processing the slowly sampled signals, of blade deflections, say once per revolution, by the diagnostic equipment. In spite of that the reconstruction is not unique, it gives very acceptable results based on the dynamic properties of the blades. The reconstructed signals may be used for estimating a fatigue damage generated in the material of blades, as well as for more reliable forcast of their residual fatigue lives.

The method evaluates approximate time series of all blade tips deflections. It is a problem of dynamic analysis of the blade to recalculate it into time series of stresses in critical points of the blade. Should the stresses be mostly uniaxial, they may be processed by the multichannel version of the rain-flow method for decomposition of complex time series into the closed cycles. In case of multiaxial stressing, the problem of fatigue damage estimation is more complicated, because there is no unique approach developed yet in the world.

6. Acknowledgement

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