AN ESTIMATION OF THE RESIDUAL LIFE OF BLADES

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Abstract: *The paper deals with a problem of estimating the length of life of long turbine blades out of measurements of stresses during an operation. A relative damage of the blades is estimated basing on law of linear cumulation of damage due to Palmgren and Miner. Results of simulated measurements of hypothetic blades are presented.*

Key words: *turbine blades, fatigue, residual life*

1. Introduction

Reliability of turbosets for nuclear power plants is of the major importance. This is the reason, why producers of such machines devote much efforts for ensuring the highest reliability as possible. One of means to get it is a monitoring of the behaviour of long blades of rotors in low-pressure cylinders of the turbines during all operating conditions. A processing of measured data enables to estimate an amount of a cumulated damage in the blades, and hence to evaluate a level of danger of a blading fracture. The paper deals with methods of estimating the residual time to a break down.

A calculation of a damage of vibrating blades is rather complicated problem, since it depends on many parameters, like

- conditions of an operation (revolutions, transmitted power, temperature, ...)
- state of blading surface (erosion, corrosion, roughness, assembling quality, ...)
- blade design (natural frequencies and modes, fixing of blades on disc, ...)
- stress distribution, material properties, production technology, ...

Some of them are given by the design of blades, the other may be measured either in advance, or during an operation of a machine. Characteristics of a material of blades may be obtained by laboratory testing using special testing machines. Parameters of vibrations and the other operational conditions should be measured permanently during all kinds of the turbine operation.

2. Fatigue of Vibrating Blades

Blades of steam turbines are slightly damped mechanical continua, the characteristic property of which is an expressive selectivity to certain frequencies contained in the applied excitation. In consequence of it, the resulting responses are mixtures of narrow band processes with dominant frequencies close to natural ones. Every blade is in this sense a mechanical band-pass filter with infinite number of bands. Fortunately, the content of higher frequencies diminishes rather rapidly. Hence, only several lowest eigen-frequencies might be considered.

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Let us assume that

a. critical points of the modes of vibrations are mutually independent,
b. natural frequencies of blades are well separated, and
c. a damage in critical points may be cumulated linearly.

These assumptions allow to accept a concept of uniaxial fatigue for an evaluation of residual lives of blades in the critical points. In opposite case a concept of multiaxial fatigue should be used, however, its full development is not finished yet.

Fatigue properties of a material of blades belong to the basic parameters needed for any evaluation of damaging effects. Results of long-term fatigue tests under harmonic loading and zero mean stress s_m are concentrated in well known S-N curves expressing amplitudes of a harmonic stress as a function of the number of cycles of loading to the total damage. This function has a form

$$\frac{N_i}{N_c} = \left(\frac{s_c}{s_i}\right)^w , \tag{1}$$

where
- s_c substitutes a fatigue limit σ_c or τ_c,
- N_c is a number of cycles on the fatigue limit s_c,
- s_i is an amplitude of the harmonic cycle under zero mean stress s_{mi},
- N_i is a number of loading cycles of the amplitude s_i to a fracture, and
- w is an exponent.

The formula holds for a polished standard testing probe made out of the material of blades. Formally, the formula (1) remains unchanged for blades with nonzero mean stress s_{mi} and with operational quality of the blade surfaces. Only critical values s_c and N_c should be changed into ${}^n s_{cm}$ and ${}^n N_{cm}$ respectively, where

$${}^n s_{cm} = s_c \, \frac{k_V \, k_q}{\beta \left({}^n K_t\right)} \left(1 - \frac{s_m}{s_F}\right)^{k_H} \tag{2}$$

where
- s_F, k_H are material constants,
- k_V is a volume factor,
- k_q is a surface quality factor,
- $\beta\left({}^n K_t\right)$ is an effective stress concentration factor as a function of ${}^n K_t$, and
- ${}^n K_t$ is the theoretical stress concentration factor of a notched part.

Formula (2) describes the generalized Haigh's diagram (see [3]), which expresses the influence of the mean stress s_m on the fatigue limit ${}^n s_{cm}$ of a prestressed notched machine part. The stress response of a blade may have the following form:

A. **Stationary process:**

- Due to the assumption ad c, the partial damage d_i caused by the i-th stress cycle of the amplitude s_i and the mean stress s_{mi} is inverse proportional to the number of cycles N_{mi} to the fracture. Hence, it follows from the equation (1):

$$d_i = \frac{1}{N_{mi}} \left(\frac{s_i}{s_{cmi}}\right)^{w_m} , \tag{3}$$

  kde w_m is an exponent dependent on s_m.

- The damage D_o caused by all cycles during the observation time T_o equals (due to the Palmgren-Miner law of linear cumulation)

$$D_o = \sum_i d_i \tag{4}$$

- Provided this law holds perfectly, the blade should break in the most critical place after $T_L = T_o/D_o$ seconds. Due to inaccuracies of the law and a model of damage, the blade comes to the end of the fatigue life L in seconds

  $$L = k_d T_L = k_d \frac{T_o}{D_o} \ , \tag{5}$$

  where k_d is a coefficient to be determined experimentally.

  If the response process were strictly stationary, one measurement would be sufficient for an estimation of the fatigue life of the blade. Unfortunately, blades in turbines operate in nonstationary regimes caused by changes of above mentioned parameters.

B. **Nonstationary process**

In this case, it is impossible to estimate the fatigue life out of one measurement. It is necessary to measure stress responses either permanently, or periodically, during all operating conditions.

Let the total time of observation T_o be composed out of n_o time intervals of measurements, every of which takes T_{on} seconds. Let the measurement covers all operating regimes of the machine, damaging or non-damaging, proportionally. Then the total life-time of the blade in seconds may be expressed by the following formula:

$$L = k_d \frac{T_o}{\sum_{n_o} T_{on}} \sum_{n_o} \frac{T_{on}}{D_{on}} \tag{6}$$

Hence, the residual life of the blade is

$$L_r = L - T_o = T_o \left(\frac{k_d}{\sum_{n_o} T_{on}} \sum_{n_o} \frac{T_{on}}{D_{on}} - 1 \right) \tag{7}$$

Now, it is rather easy to describe a strategy of the measurement and data processing:

- It is desirable to measure periodically in fixed time intervals T_{on}. In order to suppress the dead times spent by waiting for data handling, it si advisable to apply “double-buffering” in data acquisition with simultaneous data processing.
- The sampling frequency of the ideal measurement should be about by one order higher than the highest frequency in the response spectra. In this case, the local extremes of the response may be obtained with a small error, just like the signal frequency. The processing of extremes follows the above given formulae.
- Another way of measurement is based on a stroboscopic phenomenon. The measurement is much more complicated, but it may give an information on all blades of the measured disc. In this case, two additional conditions should be fulfilled:

  a. An approximate frequency of vibration should be known, and
  b. a process is stationary within the measured time T_{on}.

  Then the average amplitudes and frequencies may be evaluated out of the measured data by different methods rather reliably. One of them, published in [1] will be used also in the next chapter.

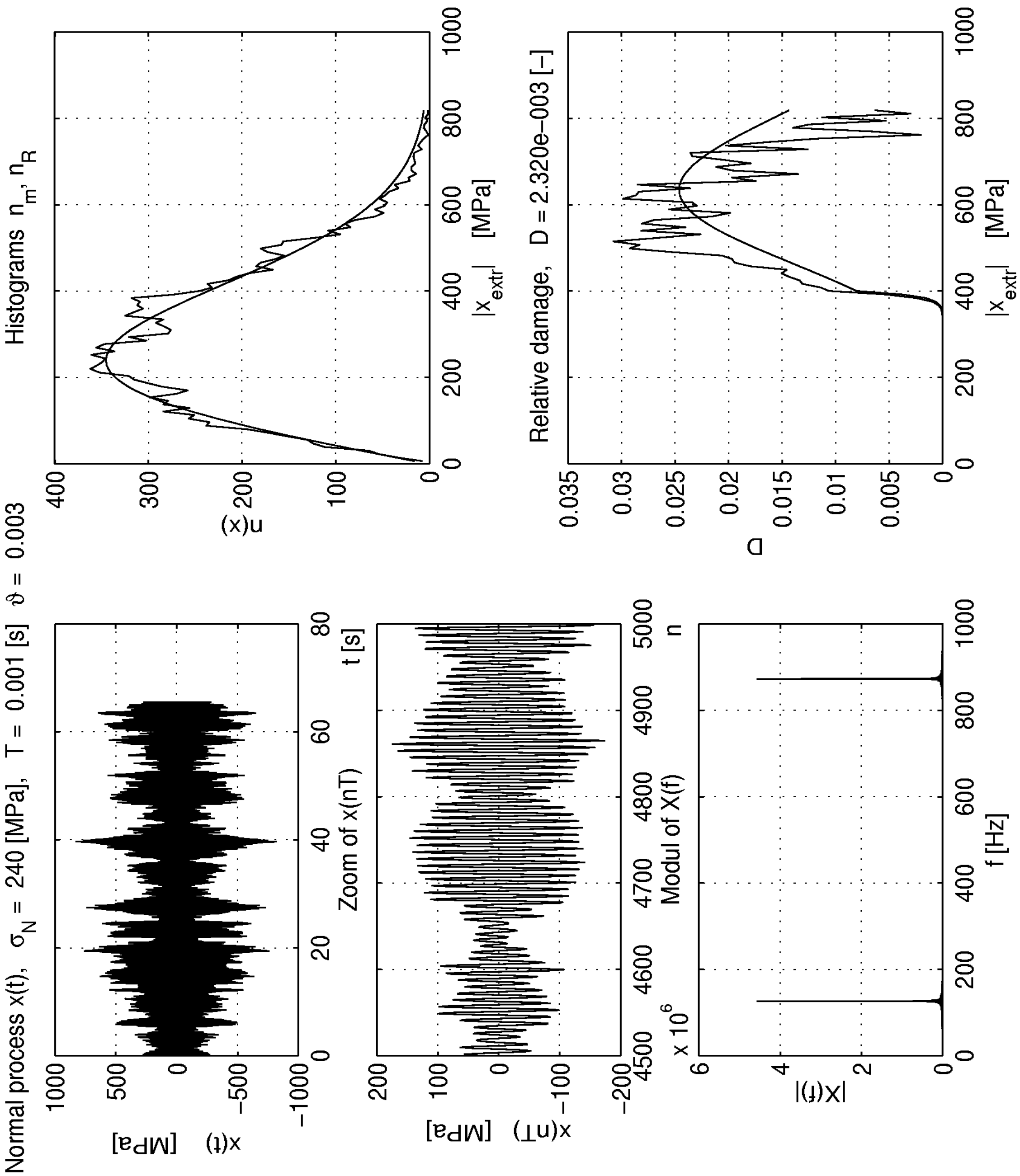

Figure 1: Simulated narrow-band process

- It is useless to store rough data for later processing on a disc, because it will be filled in a short time. This means that any data reduction process should be used before storing results. Those results may be either

  – amplitudes and frequencies in intervals T_{on}, or
  – histograms of extremes for several frequencies, or
  – cumulated damages in critical places.

3. Simulated Measurements and Data Processing

A hypothetical model of a blade rotating with a constant speed has been accepted for mor detailed investigation of different parameters. The model has been programmed in MATLAB 5.2 on PC Pentium 200 MHz. All important parameters of the model may be changed during start-up of the simulation run. The following steps in building the model were passed:

- A blade has been approximated by a slightly damped single-degree-of-freedom system with the blade eigen-frequency. Preset values, which may be changed, are $m = 1\,\mathrm{kg}$ $k = 636750\,\mathrm{N}$ giving the natural frequency 127 Hz. A damping of the blade has been preset by the logarithmic decrement $\vartheta = 0.003$.
- The external excitation is not known, however, the resulting response should be a narrow-band process. This was the reason, why the excitation has been simulated by a normal frequency limited white noise. Preset cut-off frequency $f_h = 250\,\mathrm{Hz}$ may also be changed. Two approaches to the sampling and data processing have been investigated:

Fast sampling

- It is desirable to have the sampling frequency f_s as high as possible, but at least $f_s = 2\,f_h$, where f_h is the highest frequency in spectra. The program offers $f_s = 1000\,\mathrm{Hz}$. A length of a realization of the process is free, but it is reasonable to have it as long as possible. It is advisible to have N as a power of 2 in order to use the advantage of FFT. The preset value has been $N = 2^{16} = 65536$ samples.
- The response to an external excitation has the normal distribution with the probability density function

$$f_G(x) = \frac{1}{\sigma_G\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu_x}{\sigma_G}\right)^2\right] \tag{8}$$

  where $\mu_x = \mathsf{E}\{x\}$ is a mean and σ_G is a standard deviation of x, respectively. The response has been obtained by convolving the input frequency-limited white noise by a discrete frequency response of the SDOF system, and scaled to the required σ_G. The resulting stress response is the zero mean narrow-band gaussian random proces, with the Rayleigh's probability density of magnitudes of local extremes $Y = |x_{extr}|$

$$f_R(Y) = \frac{Y}{\sigma_G^2} \exp\left[-\frac{1}{2}\left(\frac{Y}{\sigma_G}\right)^2\right] \tag{9}$$

- The Fig. 1 shows the generated process $x(t)$ of duration 65.536 seconds, a zoom of it within the time interval between 4.5 and 5 seconds, Fourier transform of $x(t)$, histograms of extremes, and the corresponding relative damages, respectively. Some parameters of the simulation run are given in the picture headings.
- Local extremes of $x(t)$ have been found by means of the single MATLAB statement

```
y = xi((xi>x(im)&xi>x(ip)) | (xi<x(im)&xi<x(ip))); ,
```

  where vector `xi=x(i)`, where `i=2:N-1, ip=i+1, im=i-1`. The extremes found were classified into the histogram. Class frequencies, centers of 100 class intervals and histogram plot have been made by the of standard MATLAB function `hist`. The histogram of the Rayleigh's distribution with the same standard deviation σ_G has been drawn into the same plot (smooth curve).

- The histograms have been used for the evaluation of the blade damage. For the purpose, a hypothetic material has been used possessing the following properties:

fatigue limit	s_{cm}	$= 400$ [MPa]
critical number of cycles	N_{cm}	$= 5e6$
exponent for $N_i < N_{cm}$	w_m	$= 6$
exponent for $N_i > N_{cm}$	$w_m^{'}$	$= w_m\,(1 + k_w)$
raise factor	k_w	$= 6$

It is clear, that the S-N curve has been chosen as piece-wise linear in log-log coordinates with the break point at $(N_{cm},\, s_{cm})$. All extremes have damaging effects in this case.

Slow sampling

The first figure shows that the process of the response is locally nonstationary. If the amplitude modulation is rather fast, the time sub-intervals to be processed should be short in order to evaluate a reliable estimate of mean amplitude of a harmonic process. There are at least two effects observed, when sampling with low frequency:

a. a stroboscopic effect transposing real frequencies below Nyquist frequency, $f_s/2$,
b. an absence of expressed extremes in short time intervals.

Both effects were investigated elsewhere (see [1]). Let us present here an improved version of the method, which is based on the $\mathcal{Z}$-transform of a harmonic signal $x(t) = x\, cos(2\pi f_a t)$. If we denote $2\pi f_a/f_s = 2\pi f_1 T$ as Ω, we may derive that

$$\cos\,\Omega \approx \frac{(\boldsymbol{x}_{n-1} + \boldsymbol{x}_{n+1})^T\, \boldsymbol{x}_n}{2\, \boldsymbol{x}_n^T\, \boldsymbol{x}_n}\ , \tag{10}$$

where $\boldsymbol{x}_n$ is a column vector of the internal samples (for $2 \le n \le N-1$). Hence, the frequency of the harmonic signal may be easily calculated as

$$f_a = \frac{1}{2\pi T}\, \mathrm{arc}\cos\,(\cos\,\Omega) \tag{11}$$

Similarly, a mean amplitude of the harmonic signal is determined by the formula

$$X \approx \sqrt{\frac{\boldsymbol{x}_n^T\, \boldsymbol{x}_n - 2\, \boldsymbol{x}_n^T\, \boldsymbol{x}_{n+1} \cos\Omega + \boldsymbol{x}_{n+1}^T\, \boldsymbol{x}_{n+1}}{(N-2)\,(1-\cos^2\Omega)}} \tag{12}$$

The effects of the length of sub-intervals and of the sampling frequency f_s have been studied in the simulation run.

n_s	n_p	T_{ef} [s]	$f_{s,ef}$ [Hz]
1	20	20/1000	50
2	10	10/1000	100
4	5	5/1000	200
5	4	4/1000	250
10	2	2/1000	500
20	1	1/1000	1000

- The same data gained by the high-speed sampling of 1000 Hz have been used for slow sampling as well. Only the total series of samples had to be "decimated". Let the symbol n_s represents the number of pickups distributed uniformly along the stator circumference, and the symbol n_p, a subscript increment in the original time series for a sampling synchronized with the rotation of a machine (see the table). A slow sampling brings the very serious problem called "aliasing". If the frequency of a blade is expected to be $f_n = 127\,\text{Hz}$, the aliased frequency is $f_a = 23\,\text{Hz}$, when sampling once per revolution with one sensor only (the sampling frequency is $fs = 50\,\text{Hz}$). Hence, the user should know the expected frequency in advance, and the evaluation of the equation (11) may be used only for calculating the aliased frequency f_a. The frequency of a blade f_n should be determined by correcting f_a.

- The influence of sampling frequency has been tested by a simulation run. The results are presented in the Fig. 2.

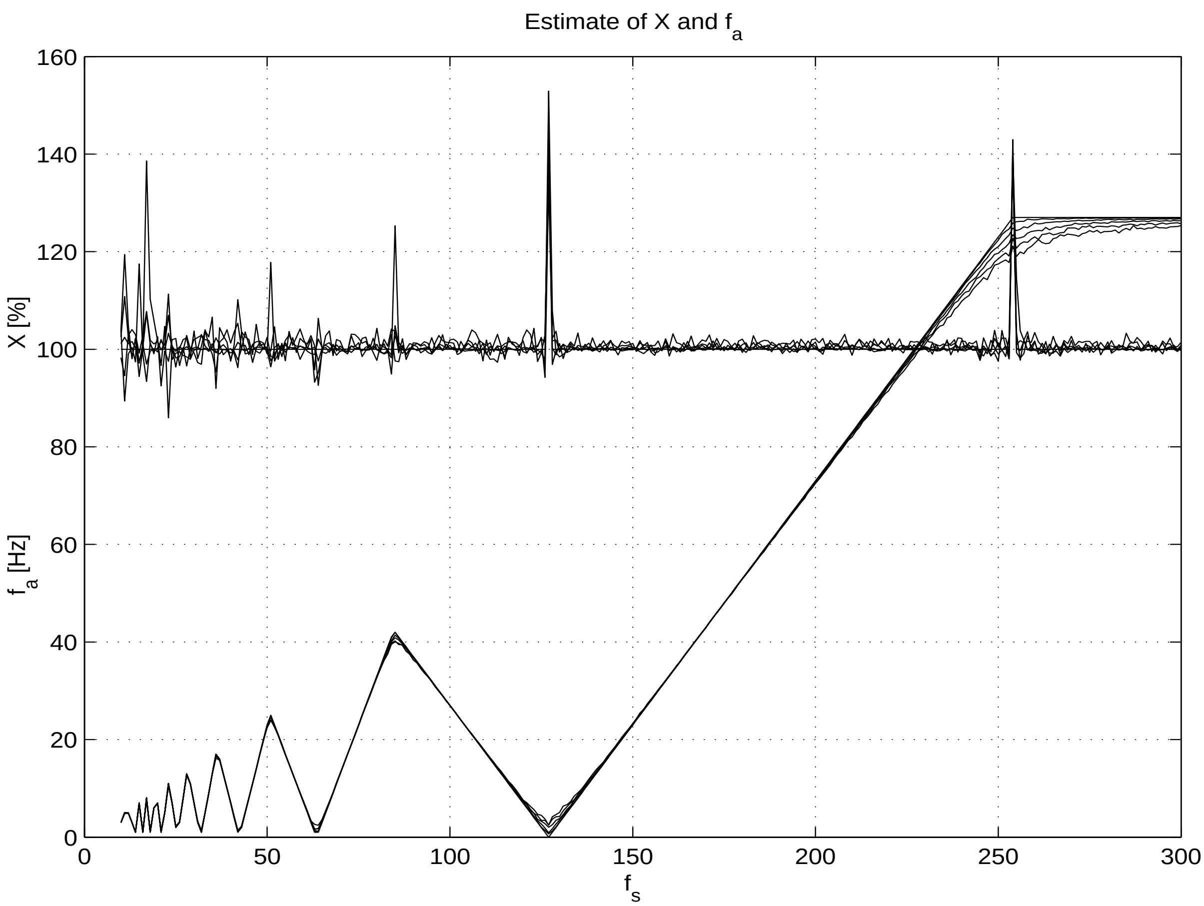

Figure 2: Aliased frequency f_a versus sampling frequency f_s

It contains simulation results of the analysis of a harmonic signal of $f_n = 127\,\text{Hz}$ and $X = 100$ with an additional normal noise of RMS $\sigma_G = 0, 2, ..., 10\%$. It shows that the aliased frequencies f_a are V-shaped functions of the sampling frequency f_s in frequency ranges containing a fractional signal frequency f_n/k, $k = 1, 2, ...$ In the k-th frequency range, it holds

$$f_n = k\, f_s \mp f_a \;, \tag{13}$$

where minus holds for the right-hand side branch of the function, and plus for the other. Analyzing this formula in adjoining ranges, the peak aliased frequencies f_{ax} and corresponding sampling frequencies f_{sx} are

$$f_{ax} = \frac{1}{2\,k+1}\, f_n \qquad \text{and} \qquad f_{sx} = \frac{2}{2\,k+1}\, f_n. \tag{14}$$

An estimate of the signal frequency is then

$$f_n = k\, f_s + f_a\, [1 - 2\, (f_s > f_n/k)] \tag{15}$$

The conditional expression in the above formula evaluates as 1 if the condition is true, or 0 otherwise. Only a coarse value of f_n is sufficient as an input quantity.

4. Conclusions

Two different approaches to the processing of vibration data for predicting a residual fatigue life have been presented, and give the following conclusions:

- The method of fast sampling uses sensors attached to selected blades. No information on behaviour of other blades are at disposal. The estimated frequencies are undisturbed.
- The method of slow sampling uses sensors attached to a stator of a machine. It may afford the information on all blades of the wheel. An approximate knowledge of a narrow-band signal of vibration should be known in advance. The method gives more precision to it.
- The proposed method of processing data based on $\mathcal{Z}$-transform proved to be applicable for evaluating mean amplitudes and frequencies in short sub-intervals of the time series, what is necessary for reliable estimating the residual fatigue-life of blading.
- If the amplitude of vibrations is changing expressively, and the sub-intervals of processing are long, the residual life may be overestimated even by several orders. Hence, the sub-intervals should be chosen appropriate with respect to a "frequency" of the amplitude modulation.
- It is necessary to avoid the case, in which the sampling frequency almost coincides with multiples of the Nyquist frequency $f_n = f_s/2$, or with natural fractions of the signal frequency f_n. In those cases the errors both of the amplitude and the frequency estimates are extremely high. This feature is characteristic for all methods, the proposed one included.

5. Acknowledgement

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6. References

[1] Balda M.: An estimation of parameters of the harmonic signal out of its samples (In Czech). Proc. of the Symposium "Computational Mechanics '98", Pernink, 1998

[2] Balda M.: An evaluation of a damage (In Czech). Private Communication, Institute of Thermomechanics of the Czech Academy of Sciences, Plzeň, 1998

[3] Balda M.: Prediction of Damage Cumulation in Vibrating Rotors. Proc. of IFToMM Fifth International Conference on Rotor Dynamics, Darmstadt, 1998, pp.