IDENTIFICATION OF LARGE-SCALE STRUCTURES

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1. Introduction

A reliability of any technical system becomes a necessity in conditions of the hard competition on the market. In case the system is nuclear power plant, a requirement of the total reliability must be fulfilled without any exception. This is the reason why high attention is paid to initial measurements of dynamic properties of primary circuits of nuclear power plants. The measurements should serve as a reference for the future operational diagnostics.

It has to be stressed out that there are software products on the market, which may solve the problem of processing such complicated experiments. However, there have been several serious reasons, why this approach has not been acceptable for processing data obtained by vibration testing in a nuclear power plant of the type VVER 440. Firstly, the price of such software is extremely high. Secondly, the bought system is a closed "black box", which you may use like "as is", but have only a little power to adjust the properties as you like. Thirdly, the applied methods for the identification need not be the optimal ones for a given task and data. All those facts initiated a development of own method, the description of which follows.

2. Identification of linear discrete systems

Let us have a linear system with multiple inputs and multiple outputs (MIMO system) to be identified. The data are frequency responses in all measured places due to the excitation in all required points. These frequency responses were evaluated out of time series of all inputs and outputs. The scheme of such a system is in Fig. (1).

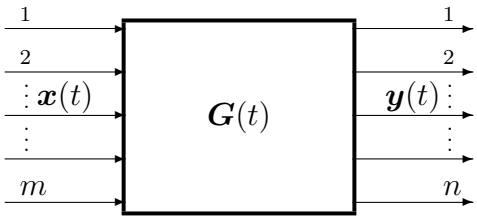

Figure 1: MIMO system

Let us assume that its behaviour is described by the ordinary differential equation (1) in the standard notation

$$\boldsymbol{M}\,\ddot{\boldsymbol{q}}(t) + \boldsymbol{B}\,\dot{\boldsymbol{q}}(t) + \boldsymbol{K}\,\boldsymbol{q}(t) = \boldsymbol{f}(t) \qquad (1)$$

It should be borne in mind that all samples $\boldsymbol{x}(kT) = \boldsymbol{f}(kT)$ and $\boldsymbol{y}(kT) = \boldsymbol{q}(kT)$ of measured signals are superposed by an unknown noise signals, the presence

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of which has injured the frequency response matrix $\boldsymbol{G}(p) = \boldsymbol{Y}^{-1}(p)\,\boldsymbol{X}(p)$ by an error matrix $\Delta\boldsymbol{G}(p)$ into a matrix $\tilde{\boldsymbol{G}}(p)$, which is obtained from the Fourier transforms $\tilde{\boldsymbol{X}}(p)$ of inputs with noise and $\tilde{\boldsymbol{Y}}(p)$ of outputs with noise as

$$\tilde{\boldsymbol{G}}(p) \;=\; \tilde{\boldsymbol{Y}}(p)^{-1}\,\tilde{\boldsymbol{X}}(p) \;=\; \boldsymbol{G}(p) + \Delta\boldsymbol{G}(p)\,, \tag{2}$$

where $p = i\omega = i2\pi f$ is the frequency parameter and $\Delta\boldsymbol{G}(p)$ a matrix of an additional noise effect. It has been derived elsewhere (see [1]) that the frequency response matrix $\boldsymbol{G}(p)$ may be expressed through the spectral matrix $\boldsymbol{S}$ and modal matrices $\boldsymbol{V}_q$ and $\boldsymbol{W}_q$ of the measured system as

$$\boldsymbol{G}(p) \;=\; \boldsymbol{V}_q\,(\,p\boldsymbol{I} - \boldsymbol{S}\,)^{-1}\boldsymbol{W}_q^H \tag{3}$$

Should the influence of the $\Delta\boldsymbol{G}(p)$ be negligible, $\boldsymbol{G}(p)$ might serve to the direct evaluation of the unknown spectral and modal matrices, as is presented in the subsection 2.3. Unfortunately, this use to be not the case of real measurements. As usual, $\Delta\boldsymbol{G}(p)$ influences $\boldsymbol{G}(p)$ so much that the global identification procedure `Gt2SVW` described in [1] yields completely wrong results. In order to avoid such a behaviour of the identification process, the measured data should be "cured" by some kind of filtering, the role of which is to suppress the influence of $\Delta\boldsymbol{G}(p)$. This might be reached by several ways, however, the most natural one seems to be the way of partial identification of every element of the matrix $\boldsymbol{G}(p)$ or $\tilde{\boldsymbol{G}}(p)$ as a SISO system. The identified element $\hat{\boldsymbol{g}}_{ij}(p)$ does not contain strong influence of the noisy $\Delta\boldsymbol{G}(p)$ any more. As soon as all elements of $\hat{\boldsymbol{G}}(p)$ are reconstructed, the matrix $\hat{\boldsymbol{G}}(p)$ may be used for the identification of the whole system.

2.1 Localization of resonant frequencies

The first step in many identification methods is to find approximate positions of natural frequencies with respect to the excitation frequencies. The equation (3) may serve as a starting one for the derivation of a formula for a frequency response of a SISO system associated with an i-th point of measurement and a j-th point of excitation:

$$\hat{g}_{ij}(p) \;=\; \boldsymbol{v}_i^T\,(\,p\boldsymbol{I} - \boldsymbol{S}\,)^{-1}\,\boldsymbol{w}_j^C \;=\; \sum_\nu \frac{\boldsymbol{v}_{i\nu}^T\,\boldsymbol{w}_{j\nu}^C}{p - s_\nu} \;=\; \sum_\nu \frac{a_{ij\nu}}{p - s_\nu}\,, \tag{4}$$

where $\boldsymbol{v}_i^T$ is the i-th row of the matrix $\boldsymbol{V}_q$, $\boldsymbol{w}_j^C$ is the j-th column of the matrix $\boldsymbol{W}_q^H$, and s_ν is the ν-th eigenvalue of the system. It is visible from the equation (4) that the measured frequency responses contain contributions of all modes. More over, modules of $\tilde{g}_{ij}(p)$ will be well pronounced in the vicinity of eigenvalues s_ν, because they are inversely proportional to the distance of exciting parameter p from s_ν. Since the natural frequencies belong to the properties of the whole system, the influence of them should be contained in all elements of $\tilde{g}_{ij}(p)$. Hence, it is good idea to sum all modules of $\tilde{g}_{ij}(p)$ for suppressing random errors and amplifying the information on position of natural frequencies out of peaks of $\sum_i \sum_j |\tilde{g}_{ij}(p)|$.

It was found very early, that the "total amplitude characteristics" in displacements gives good information on the lowest resonance peaks, while the highest are attenuated. The opposite occurs, when working with frequency responses in acceleration. The best results were obtained, when summing absolute values of frequency responses in velocities, since there is good chance to observe peaks of the function – mobility

$$g_v(p) \;=\; \sum_i \sum_j |\,p\,\tilde{g}_{ij}(p)| \tag{5}$$

in the whole frequency interval with equal sensitivity. They give approximate values of natural frequencies for slightly damped systems.

Not all peaks may be used for the identification, because some of them are caused by measuring errors or by a resonance of a single unimportant part of the system. Thus a relative height of the peak with respect to its neighborhood speaks about its importance.

2.2 Identification of SISO systems

The key problem in the identification of $\hat{g}_{ij}(p)$ is finding its unknown parameters $a_{ij\nu}$ and s_ν. Kozánek published a method for for solving the problem years ago [2]. The parameters were sought as a solution of a least-squares problem by minimization of the positive function

$$f(\boldsymbol{c}_{ij}, \boldsymbol{p}) \;=\; \boldsymbol{r}^H \boldsymbol{r} \;=\; \sum_k |r_k|^2 \tag{6}$$

where $\boldsymbol{c}_{ij} = [\boldsymbol{s}^T, \boldsymbol{a}^T]_{ij}^T$ is a vector of unknowns, $\boldsymbol{p} = [p_k]$ is a vector of exciting frequencies, and r_{ijk} in an ij position in the matrix is a residuum

$$r_{ijk} \;=\; \hat{g}_{ij}(p_k) - \tilde{g}_{ij}(p_k) \tag{7}$$

Unfortunately, the original method is rather sensitive to measuring errors and as such unusable for the application to real data. This was the reason, why we have reconstructed it completely.

The form of the equation insinuates that the method of solution should be iterative, because unknowns s_ν are in denominators of the fractions. The unknowns create a vector $\boldsymbol{c}$ for the ij-th element of the matrix $\hat{\boldsymbol{G}}(p)$, where

$$\boldsymbol{c}_{ij}^T \;=\; [\, s_1, \cdots, s_\nu, \cdots, s_n,\, a_1, \cdots, a_\nu, \cdots, a_n \,]_{ij} \;=\; [\, \boldsymbol{s}^T,\, \boldsymbol{a}^T \,]_{ij} \tag{8}$$

The unknowns $\boldsymbol{c}$ may be solved, say, by the Newton-Raphson method, out of an over-determined system of nonlinear equations as a least-squares problem in iterations (with subscripts ij omitted):

$$\boldsymbol{c}^{(l+1)} \;=\; \boldsymbol{c}^{(l)} - \boldsymbol{J}^{(l)+}\, \boldsymbol{r}^{(l)} \tag{9}$$

where $\boldsymbol{J}^{(l)+}$ is Moore-Penrose pseudoinverse of a Jacobian matrix in l-th iteration. It has the form

$$\boldsymbol{J}^{(l)} \;=\; \frac{\partial f(\boldsymbol{c}_{ij}, \boldsymbol{p})}{\partial \boldsymbol{c}^{(l)}} \tag{10}$$

Analyzing the form of $\boldsymbol{J}^{(l)}$, we find that

$$\boldsymbol{J}^{(l)} \;=\; [\, \boldsymbol{A}_2 \operatorname{diag} \boldsymbol{a},\; \boldsymbol{A}_1 \,]^{(l)}\,, \tag{11}$$

where matrix

$$\boldsymbol{A}_1 \;=\; \begin{bmatrix} \dfrac{1}{p_1 - s_{\nu_a - 1}} & \cdots & \dfrac{1}{p_1 - s_{\nu_b + 1}} \\ \vdots & , \quad , & \vdots \\ \dfrac{1}{p_K - s_{\nu_a - 1}} & \cdots & \dfrac{1}{p_K - s_{\nu_b + 1}} \end{bmatrix}, \tag{12}$$

and the matrix $\boldsymbol{A}_2$ is composed of the squared elements of the matrix $\boldsymbol{A}_1$. Subscripts

ν_a and ν_b belong to the lowest and highest eigenvalues in the required frequency interval respectively. Including fictive natural frequencies s_{ν_a-1} and s_{ν_b+1} among the sought natural frequencies should ensure a compensation of effects of those natural frequencies laying outside the measuring frequency interval.

A good approximation of initial values of unknowns is very important for starting the iterative process. Should the eigenvalues be well separated, the equation (4) might be approximated by a single mode in the vicinity of the resonance frequency:

$$g(p) \;\doteq\; \frac{a_\nu}{p-s_\nu} + g_\nu \qquad \Longrightarrow \qquad p \;=\; s_\nu + \frac{a_\nu}{g(p)} + \frac{p-s_\nu}{g(p)}\, g_\nu \,, \tag{13}$$

where g_ν is an effective offset of a given mode from the origin in the complex plane of $\hat{g}_{ij}(p)$. It is valid for the initial guess only. The second of equations (13) may serve for a derivation of a set of equations for initial estimates. Its solution in the least-square sense is as follows

$$\begin{bmatrix} \hat{s}_\nu \\ \hat{a}_\nu \\ \hat{g}_\nu \end{bmatrix} \;=\; \begin{bmatrix} 1\,, & \dfrac{1}{g(p_{m-q})}\,, & \dfrac{p_{m-q}-s_\nu}{g(p_{m-q})} \\ \vdots & \vdots & \vdots \\ 1\,, & \dfrac{1}{g(p_{m+q})}\,, & \dfrac{p_{m+q}-s_\nu}{g(p_{m+q})} \end{bmatrix}^{+} \begin{bmatrix} p_{m-q} \\ \vdots \\ p_{m+q} \end{bmatrix} \tag{14}$$

The frequency p_m is the peak frequency obtained out of the total mobility as described above. The subscript q determines the number of values of $\tilde{g}_{ij}(p)$ on each side of p_m accepted for the mode evaluation. We use $q = 2$ as normal, what means that the eigenvalue $\hat{s}_\nu$ is estimated out of 5 points of measured frequency response, the peak $\tilde{g}_{ij}(p_m)$ as the middle one.

2.3 Identification of MIMO systems

As soon as all elements of measured $\tilde{\boldsymbol{G}}(p)$ have been identified, new matrix $\hat{\boldsymbol{G}}(p)$ may be assembled for the same frequencies as measured ones. This new matrix frequency series may serve for an global identification purposes.

The title problem was dealt with on this conference a year ago [1]. The matrix frequency series $\hat{\boldsymbol{G}}(p_k),\ k = 1,\, 2,\, \cdots, K$ was transformed into the matrix of impulse responses $\hat{\boldsymbol{G}}(t_k)$ by the discrete version of the inverse Fourier transform (IDFT). A slightly modified algorithm finds the eigenvalues of the measured system from the eigenvalue problem

$$\boldsymbol{A}\,\boldsymbol{E} \;=\; \boldsymbol{E}\,\boldsymbol{Z} \tag{15}$$

where the system matrix is

$$\boldsymbol{A} \;=\; \begin{bmatrix} \boldsymbol{F}_1^H\,, & \boldsymbol{F}_2^H\,, & \cdots\,, & \boldsymbol{F}_{p-1}^H\,, & \boldsymbol{F}_p^H \\ \boldsymbol{I}_n\,, & \boldsymbol{O}_n\,, & \cdots\,, & \cdots\,, & \boldsymbol{O}_n \\ \boldsymbol{O}_n\,, & \boldsymbol{I}_n\,, & \ddots\,, & \boldsymbol{O}_n\,, & \vdots \\ \vdots\,, & & \ddots\,, & \ddots\,, & \vdots \\ \boldsymbol{O}_n\,, & \boldsymbol{O}_n\,, & & \boldsymbol{I}_n\,, & \boldsymbol{O}_n \end{bmatrix}, \tag{16}$$

and modal matrix

$$\boldsymbol{E} \;=\; \begin{bmatrix} \boldsymbol{E}_p \\ \boldsymbol{E}_{p-1} \\ \vdots \\ \boldsymbol{E}_2 \\ \boldsymbol{E}_1 \end{bmatrix} \qquad \text{where} \qquad \begin{array}{l} \boldsymbol{E}_k \;\; = \boldsymbol{W}\, \exp^H(k\boldsymbol{S}T) \qquad \text{and} \\ \boldsymbol{E}_{k+1} = \boldsymbol{E}_k\, \exp^H(\boldsymbol{S}T) \;=\; \boldsymbol{E}_k\, \boldsymbol{Z} \end{array} \tag{17}$$

The matrices $\boldsymbol{F}_k$ in (16) are obtained as a solution of the system of linear algebraic equations

$$\begin{bmatrix} \boldsymbol{F}_1 \\ \boldsymbol{F}_2 \\ \vdots \\ \boldsymbol{F}_p \end{bmatrix} = \begin{bmatrix} \boldsymbol{G}_p & , \boldsymbol{G}_{p-1} & , \cdots & , \boldsymbol{G}_1 \\ \boldsymbol{G}_{p+1} & , \boldsymbol{G}_p & , \cdots & , \boldsymbol{G}_2 \\ \vdots & , \vdots & , & , \vdots \\ \boldsymbol{G}_{N-2} & , \boldsymbol{G}_{N-3} & , \cdots & , \boldsymbol{G}_{N-p-1} \end{bmatrix}^+ \begin{bmatrix} \boldsymbol{G}_{p+1} \\ \boldsymbol{G}_{p+2} \\ \vdots \\ \boldsymbol{G}_{N-1} \end{bmatrix} \tag{18}$$

The spectral matrix $\boldsymbol{S}$ of the measured system may be evaluated from the spectral matrix $\boldsymbol{Z}$ using the second equation of (17) as

$$\boldsymbol{S} \;= f_s \ln \boldsymbol{Z}^H \tag{19}$$

The left modal matrix $\boldsymbol{W}$ may be found rather easily out of the formula given in (17) as soon as the spectral matrix is known:

$$\boldsymbol{W} = \boldsymbol{E}_p \left[\exp^H(p\, \boldsymbol{S}T) \right]^+ \tag{20}$$

The right modal matrix $\boldsymbol{V}$ may be calculated from the equation (3) after transforming it into the form

$$\boldsymbol{V} = \boldsymbol{G}(p) \left[(p\boldsymbol{I} - \boldsymbol{S})^{-1} \boldsymbol{W}^H) \right]^+ \tag{21}$$

Last three equations yield the global identification of the observed system in spectral and modal properties. The algorithm of the global identification of MIMO systems out of the impulse response matrix has been implemented earlier and got the name `Gt2SVW`.

3 An implementation of the method

The total identification of an observed system may be done only in case that also the measurement is total, what means that all matrices of measured frequency responses $\tilde{\boldsymbol{G}}(p)$ are regularly sampled in frequency without missing data. This requirement needs the tests are perfectly prepared and performed. It is clear that such a measurement on a large scale structure with many exciting an measuring points is also time-consuming and hence expensive. This situation leads sometimes to the reduced measurements, in which only a sub-matrix of the $\tilde{\boldsymbol{G}}(p)$ is obtained. In a limiting case it is only one vector of it. This was our starting point for a development of the new programme, which has been named `IdeSIMO` (**Ide**ntification of **S**ingle **I**nput – **M**ultiple **O**utput systems).

Let us suppose a system to be identified is excited in one point and measured in many points of it. The excitation might be done by different ways (periodic, transient, random, etc.), but the sensors of movement use to be accelerometers. Hence, one column of the matrix $\tilde{\boldsymbol{G}}_a(p) = p^2\, \tilde{\boldsymbol{G}}(p)$ is found by means of the Fourier transform and equation (2). An element of the vector is a sequence of complex numbers, components of which are stored in a file,

one file per signal. A file consists of an overhead, which contains important information on the measurement, and a set of couples of real and imaginary parts of the frequency response matrix in accelerations. In order to make an assembling of the vector $\tilde{\boldsymbol{g}}_{aj}(p)$ automatic, the files should have the same structure and the self-explaining names. The whole set is placed in one single directory. The list of file names in the directory is created by the function `openset` and put into the file `list`, which is used for reading rough data files in the module `getfiles`, and later for labeling diagrams of results.

As soon as all data are read, and the vector of frequency response functions in acceleration $\boldsymbol{g}_a(p) = p^2\,\tilde{\boldsymbol{g}}_j(p)$ is ready, the total mobility function $\bar{g}_v(p) = \sum_i p\,|\tilde{\boldsymbol{g}}_{ai}(p)|$ is evaluated and its peaks found by the procedure `peaks`. This procedure finds peaks of the real function, which are greater than a neighboring valley by a prescribed value, at lest from one side. The mobility diagram with the ticks (medium size) in places of peaks is then plotted. (Fig. 2). Simultaneously the resonant frequencies are printed out and stored.

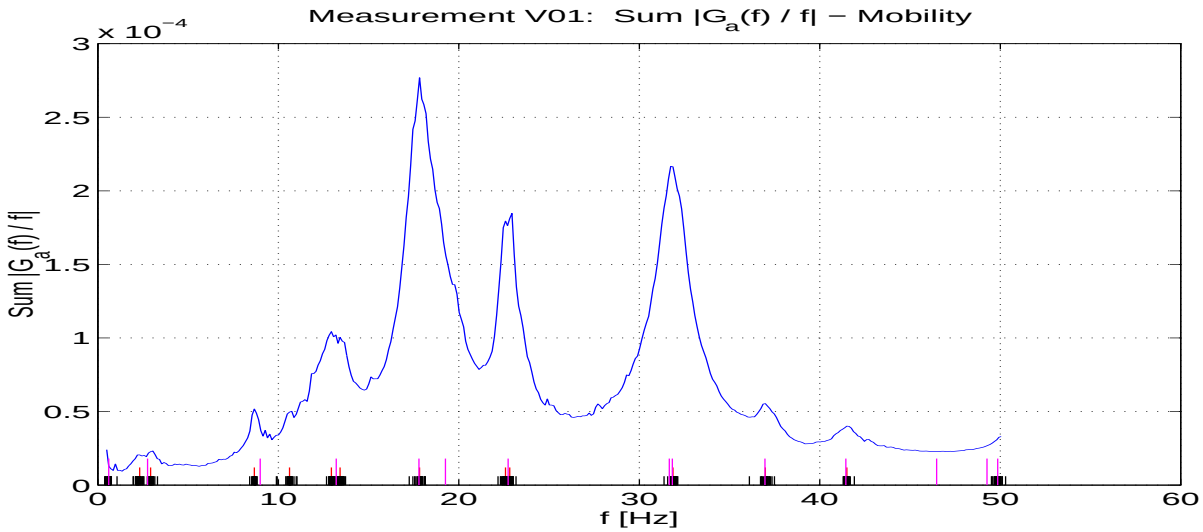

Figure 2: The mobility diagram with predicted and found natural frequencies

A user has the opportunity to chose a kind of processing data. If the data were already preprocessed as a SISO systems, it is not necessary to pass this step once more, and the programme may proceed up to the global estimation, and animation. Otherwise, the cycle of preprocessing due to the sub-section 2.2 takes place. The preprocessing is done by the function `IdeSISO`. Since it identifies the partial frequency response out of $\tilde{\boldsymbol{g}}_{ij}$, the input data should be $p^2\,\tilde{\boldsymbol{g}}_{aij}$. The procedure uses the information of expected eigenvalues from a vector of peak subscripts, estimates initial values of unknowns s_ν and a_ν, and transfers the solution of the least-square problem to the procedure `lsqnonlin` from the Optimization Toolbox of MATLAB. The solution proceeds rather fast, because the formula for the Jacobian matrix is known and implemented in the module of the goal function `funJ`, which calculates both a vector of weighted residuals and its Jacobian matrix.

The user may choose weights of measurements, as a vector of the same length as data vector. It is reasonable to choose the weights $w = f^2$ in order to balance the errors for acceleration, to which the results will be compared. The figure 3 shows the results of the SISO identification for one point of measurement.

Not every identification may give so good results. The quality of an identification depends on the relative error of measurement. If it is high, the result may be almost arbitrary.

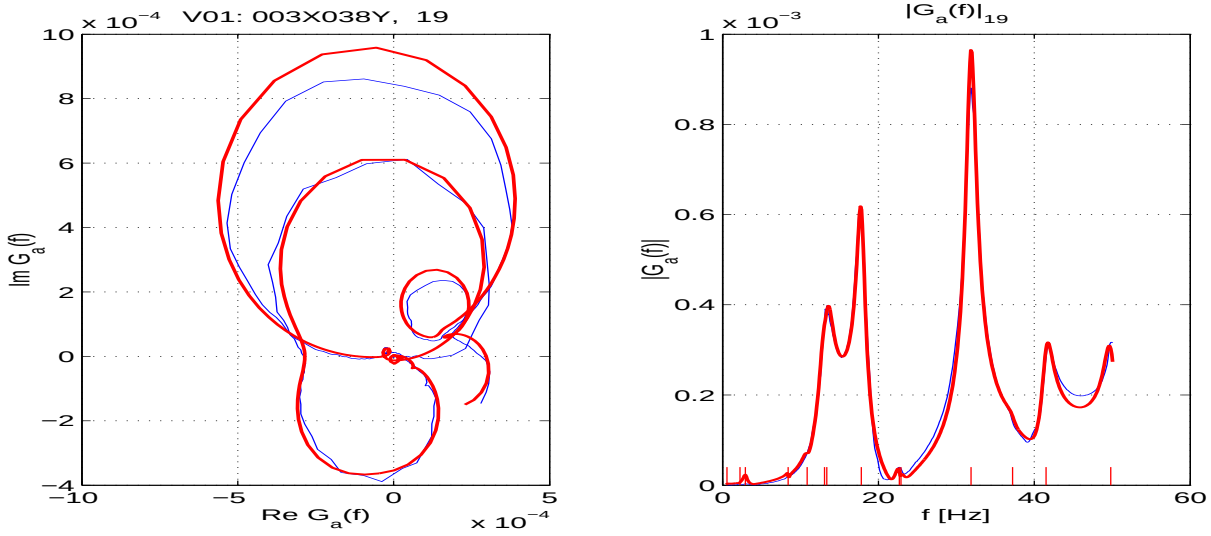

Figure 3: Measured $\tilde{\boldsymbol{g}}_a(p)$ and identified $\hat{\boldsymbol{g}}_a(p)$ frequency response in accelerations

Fortunately, high errors occurred mostly by side elements of the matrix, which had the maximum module by 2 orders lesser than that of the global maximum. The resulting smoothed $\hat{\boldsymbol{g}}_{ij}(p)$ is recorded into a matrix (vector) $\hat{\boldsymbol{G}}_j(p)$ for a further processing by `IdeSIMO`.

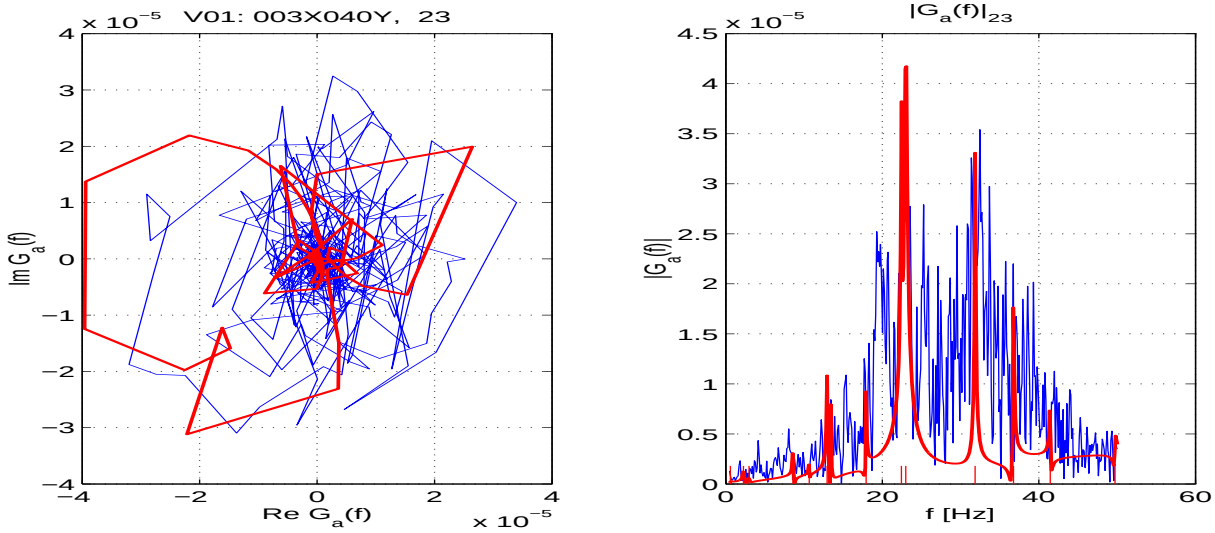

Figure 4: Measured $\tilde{\boldsymbol{g}}_a(p)$ and identified $\hat{\boldsymbol{g}}_a(p)$ with a strong noise

Every identified natural frequency is plotted in the mobility diagram as a short tick. The result of all partial identifications, here 46, is seen as heavy rectangles, which show the scattering of identified frequencies. As soon as all partial identifications are finished the matrix $\hat{\boldsymbol{G}}_j(p)$ is complete. The final global SIMO identification may start by Fourier transformation of $\hat{\boldsymbol{G}}_j(p)$ to $\hat{\boldsymbol{G}}_j(t)$, and by calling the procedure `Gt2SVW`. The results of it are output as complex natural frequencies and damping quotient (in %). The final real parts of natural frequencies are plotted again in Fig. 2 as longest ticks.

The last part of the program `IdeSIMO` animates the undamped vibration in the identified modes $\boldsymbol{V}$. It is not an essential part, nevertheless, it gives very good insight on particular

modes. It is implemented in the module `anim3D`. The geometry of the measured structure is read out of the file `xyz.txt`, which has to be prepared in forward by the user. It contains code of the measuring point and its three coordinates in the space. Next, it creates a full screen 3D view on the structure line model complemented by the visualized mode. That is updated in 252 time intervals, what corresponds to two full periods of vibration. The user has an opportunity either to repeat it, or to continue with the next or an arbitrarily chosen mode. Fig. 5 shows an example.

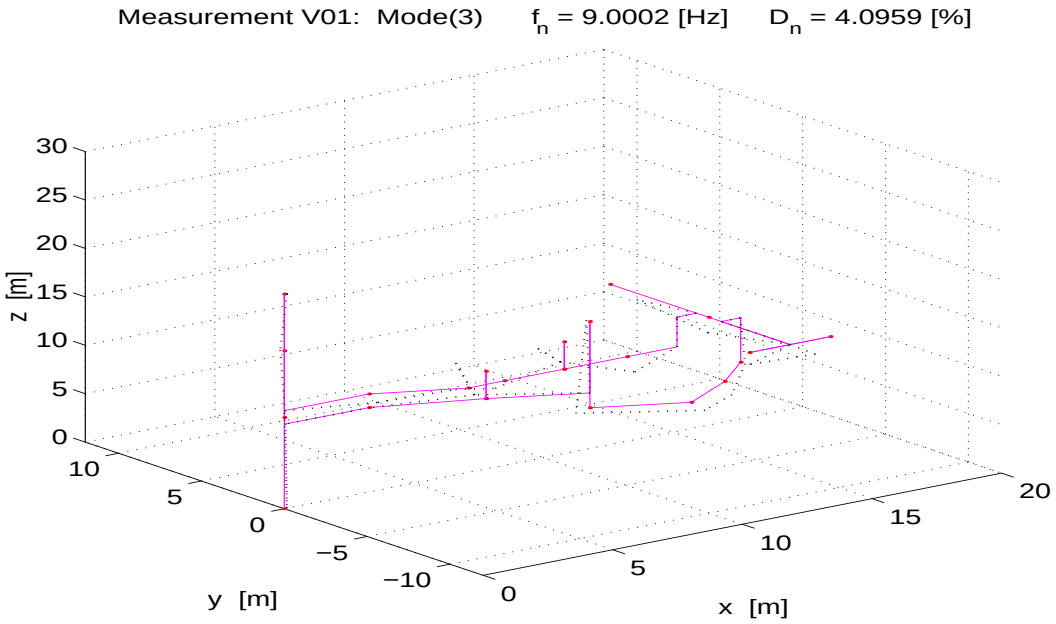

Figure 5: A natural mode animation

Conclusions

The contribution deals with a problem of an identification of big structures out of data which were obtained by a real experiment. The main ideas of the data preprocessing as a SISO systems and final global processing and animation of SIMO system are presented.

Acknowledgment

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References

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Resumè

The new approach to the global identification of large-scale structures out of noisy frequency responses in accelerations is described. The identification procedure is stabilized by filtering the experimental data by the partial identification of point reponses by SISO systems. The animation of the structure vibration in natural modes is the last stage of the whole procedure, which has been implemented in MATLAB v. 5.3.