Processing of Blade Monitoring System Data

Miroslav Balda
Academy of Sciences of The Czech Republic, Institute of Thermomechanics

1 INTRODUCTION

The total reliability of a nuclear power plant requires the same for its single parts. Rotating blades of a turbine belong to the most loaded parts of nuclear power plant equipment. This is the reason why considerable attention is paid to blade vibration during an operation of the turbine. For the purpose, special monotoring systems have been developed for following behaviour of blades in service conditions. The monitoring system should give messages when a danger of a fatigue damage occurs. It takes place in moments, when dynamic stresses in critical places of blades exceed a safe level.

2 MEASUREMENTS

New monitoring systems for blade vibrations are based on an exact time measurements (1), (2). Clock pulses of a high constant frequency f_c are counted into a counter, the contents of which are stored at the moments when tips of blades are passing sensors attached to a turbine stator. The stored sequence of the counts is a typical nonstationary time series. The nonstationarity originated by a rotation is superposed on useful signals of blade tip deflections from their equilibrium positions. Figure 1 shows a typical set of measured data in three stages of their preprocessing. There was only one sensor in the stator of the turbine, the rotor of which was running by 3000 RPM, giving the effective sampling frequency $f_s = 50$ Hz. One measurement period of the duration 2 seconds performed through one sensor gave 100 samples per blade.

In the Figure 1a and 1b, there are samples of all signals taken from 47 blades and one datum drawn after subtracting a linear trend caused by an average rotor speed. It is seen that the blade signals the thick line in the Figure 1a are still nonstationary. The thin line drawn a distance higher is the smoothed average of all blade signals. The strongly oscillating signal belongs to the datum, and corresponds the torsional vibration of a rotor.

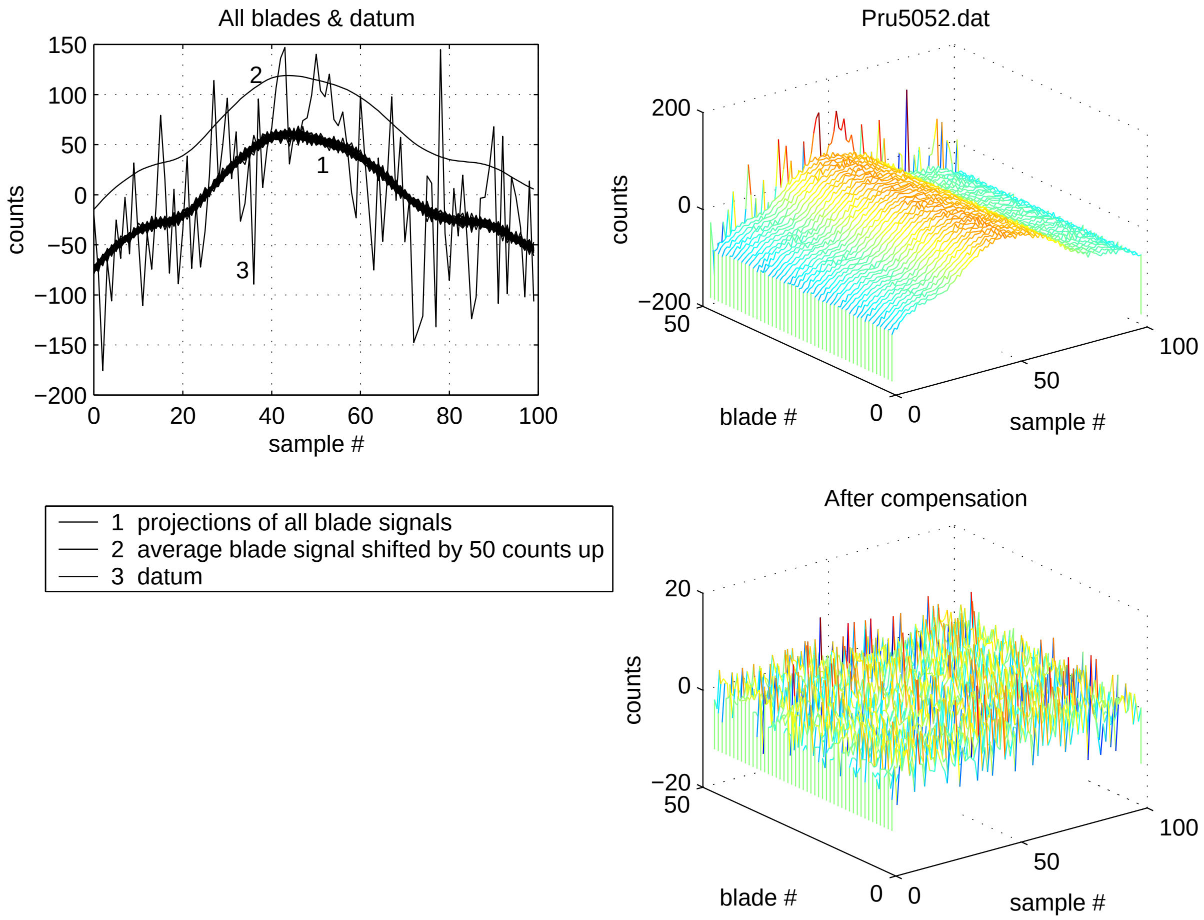

Figure 1: Measured signals of blades and a datum

3 SIGNAL RECONSTRUCTION

Instantaneous blade tip deflections may be obtained by the above mentioned preprocessing and scaling of the stored numbers of counts. A very important problem is raised by a low frequency sampling of the blade signals with respect to natural frequencies of blades. The information on frequency components of the sampled signals may give the Fourier transform.

3.1 Fourier analysis of sampled signals

Sampling the signal $h(t)$ of blade tip deflections by a sampling frequency $f_s = 1/T$, where T is the sampling period, corresponds to the multiplication of $h(t)$ by the Dirac comb

$$\delta_T(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT), \tag{1}$$

as a periodic series of Dirac pulses shifted mutually by T in time. The operation generates a time series, a sampled function

$$h_T(t) = h(t)\ \delta_T(t) = \sum_{k=-\infty}^{\infty} h(kT)\ \delta(t - kT) \qquad (2)$$

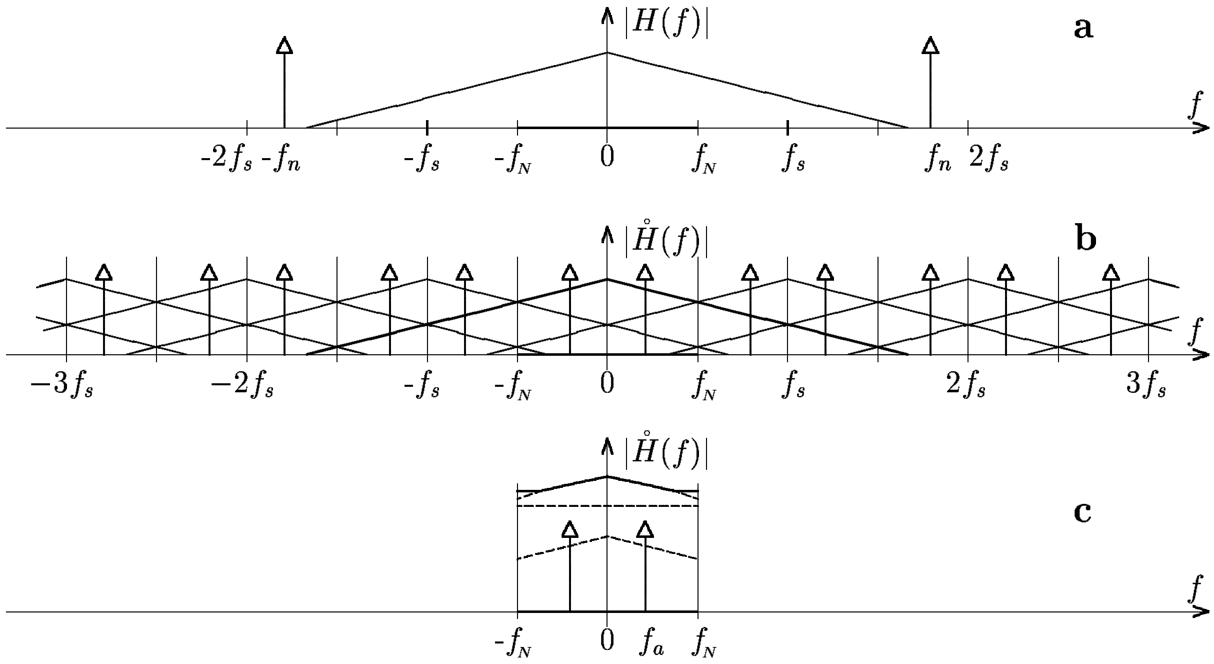

Figure 2: Aliasing produced by a low sampling frequency $f_s = 2f_N$

Since the Fourier transform (FT) of the Dirac comb

$$\mathrm{F}\{\delta_T(t)\} = \frac{1}{T}\delta_{f_s}(f) \qquad (3)$$

is a (scaled) Dirac comb in the frequency domain with the repetition period $f_s = 1/T$, the Fourier transform of the sampled signal is

$$\mathrm{F}\{h_T(t)\} = \int_{-\infty}^{\infty} h(t)\ \delta_T(t)\ e^{-i2\pi ft}dt = \int_{-\infty}^{\infty} h(t) \sum_{n=-\infty}^{\infty} \delta(t - nT)\ e^{-i2\pi ft}dt =$$

$$= \frac{1}{T}H(f) * \delta_{f_s}(f) = H_{f_s}(f) \qquad (4)$$

The formula expresses a convolution of the Fourier transforms $H(f)$ of the original function $h(t)$ and $\delta_{f_s}(f)$ of the Dirac comb $\delta_T(t)$ in the frequency domain. It is seen from equation (3) that the Fourier transform of the sampled signal is an infinite sum of all copies of the Fourier transform of the original function shifted by multiples of the sampling frequency f_s. In order to avoid any interference of the side frequency bands with the basic one in the interval $(-f_N, f_N)$, where $f_N = f_s/2$ is so called Nyquist frequency, the sampling frequency f_s should be chosen at least as a double the highest frequency f_h contained in the signal:

$$f_s \geq 2f_h \tag{5}$$

In the example from the Figure 2a, $f_h = f_n$. The formula (5) expresses the well-known Shannon sampling theorem. The convolution in formula (4) introduces very serious problem in signal processing. If the signal were undersampled, what means, if its highest frequency contained in the signal were higher than f_N, the frequency components outside the basic frequency range $(-f_N, f_N)$ would be moved to another frequency into that basic band. This phenomenon, called “aliasing” or ”folding”, is well demonstrated in the Figure 2. The low sampling frequency f_s causes a repetition of the original FT (from Figure 2a) into side bands (Figure 2b), and a summing of overlapping contributions of all bands (Figure 2c). The particular frequency component f_n is projected into the basic band to another, aliased frequency f_a, which may be evaluated using the following formula

$$f_a = f_n - f_s \,\mathrm{round}\left(\frac{f_n}{f_s}\right) = f_n - f_s k \tag{6}$$

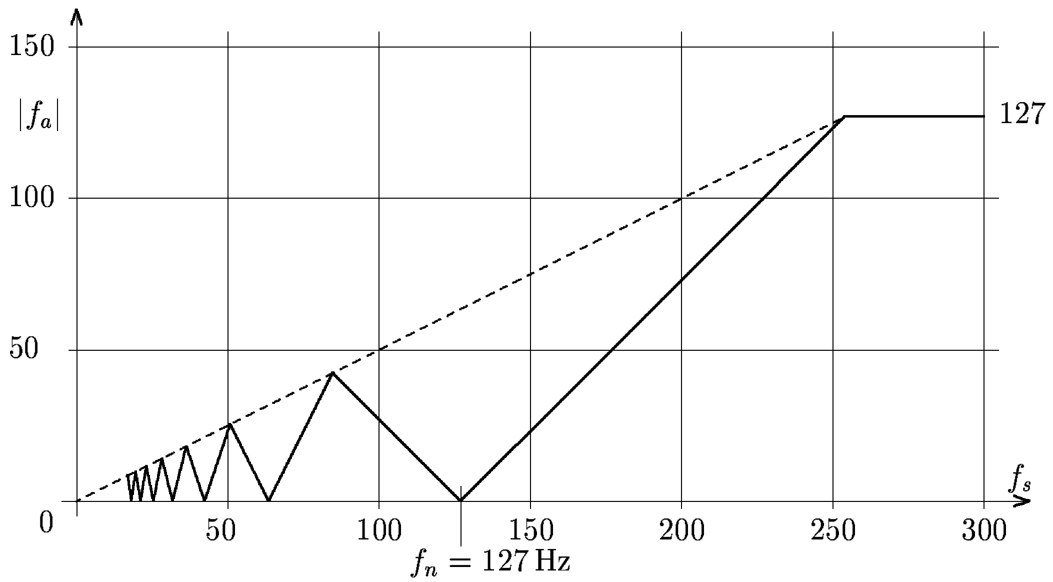

Figure 3: Aliased frequency f_a as a function of a sampling frequency f_s

The Figure 3 shows the example of aliasing the frequency $f_n = 127$ Hz, when sampled by an arbitrary sampling frequency f_s. The attainable peak aliased frequencies f_{ax} and corresponding sampling frequencies f_{sx} are

$$f_{ax} = \frac{1}{2k+1} f_n, \quad \text{and} \quad f_{sx} = \frac{2}{2k+1} f_n \quad \text{respectively.} \tag{7}$$

The signal becomes unobservable, when $f_s = f_n / n$, with natural n, for which $f_a = 0$. In this case, a particular frequency component occurs at zero frequency of an aliased spectrum.

3.2 Reconstruction of vibrating blade signals

A signal may not be reconstructed if the sampling frequency does not comply with the sampling theorem. This fact would disable, in general, any attempt to estimate a fatigue life of rotating blades, if the signals were undersampled. In that case, there would be only sparse information at disposal on peaks of stresses, which cause the fatigue damage and its cumulation. Fortunately, there is a way available, how to support an estimation of the residual life of blades. The monitored turbine disc with blades is not a black box for an observer. The monitored disc is well known from pre-operation investigations during its development. This means that estimates of the resonance frequencies $\tilde{f}_n$ are known quite well in forward from numerical and experimental analyses. The rotational speed and the number of sensors distributed uniformly along the stator circumference determine the sampling period f_s. The estimates of aliased frequencies corresponding the forecasted resonant frequencies of blades in the Fourier transform of slowly sampled signals will be

$$\tilde{f}_a = \tilde{f}_n - f_s \,\mathrm{round}\left(\frac{\tilde{f}_n}{f_s}\right) = \tilde{f}_n - f_s \tilde{k} \qquad (8)$$

It is rather easy to find peaks in the Fourier transform, which are within a distance of μ elementary bands on both sides from the estimated aliased frequencies due to Equation (8). The found peak frequencies may be recalculated to f_n using formula (6), where $\tilde{k}$ is used instead of k. The reconstruction will then be done in the following steps:

a. Choose an appropriate fictive sampling frequency $\alpha f_s > 2 f_h$, where f_h is the highest natural frequency to be identified.
b. Build a new reconstructed Fourier spectrum $H_{k f_s}(f)$ in the interval $\alpha(-f_N, f_N)/2$ by moving the parts of the aliased spectrum to places belonging to the frequencies f_n. Those parts consist of the peak complemented by ν adhering frequency components from its both sides. An attention should be paid on the way, how the aliased spectrum was originated, as far as the order and phases of components are concerned.
c. Spread a rest of the aliased Fourier spectrum over the $H_{k f_s}(f)$ due to the user requirements.
d. Apply the inverse FT to the reconstructed spectrum to obtain a reconstructed signal of the blade tip deflection

$$h_{T/k}(t) = \mathrm{F}^{-1}\left\{H_{k f_s}(f)\right\} \qquad (9)$$

It is necessary to stress that the described method of the signal reconstruction gives only one approximation out of many others. All approximations are fitting the sequence of original signal samples exactly, if all frequency components of the aliased Fourier spectrum have been properly transferred into the reconstructed one. In that case, the reconstruction yields an acceptable signal for the damage estimation. Methods that are more accurate might be developed, however, they are all much more complicated and as such time consuming.

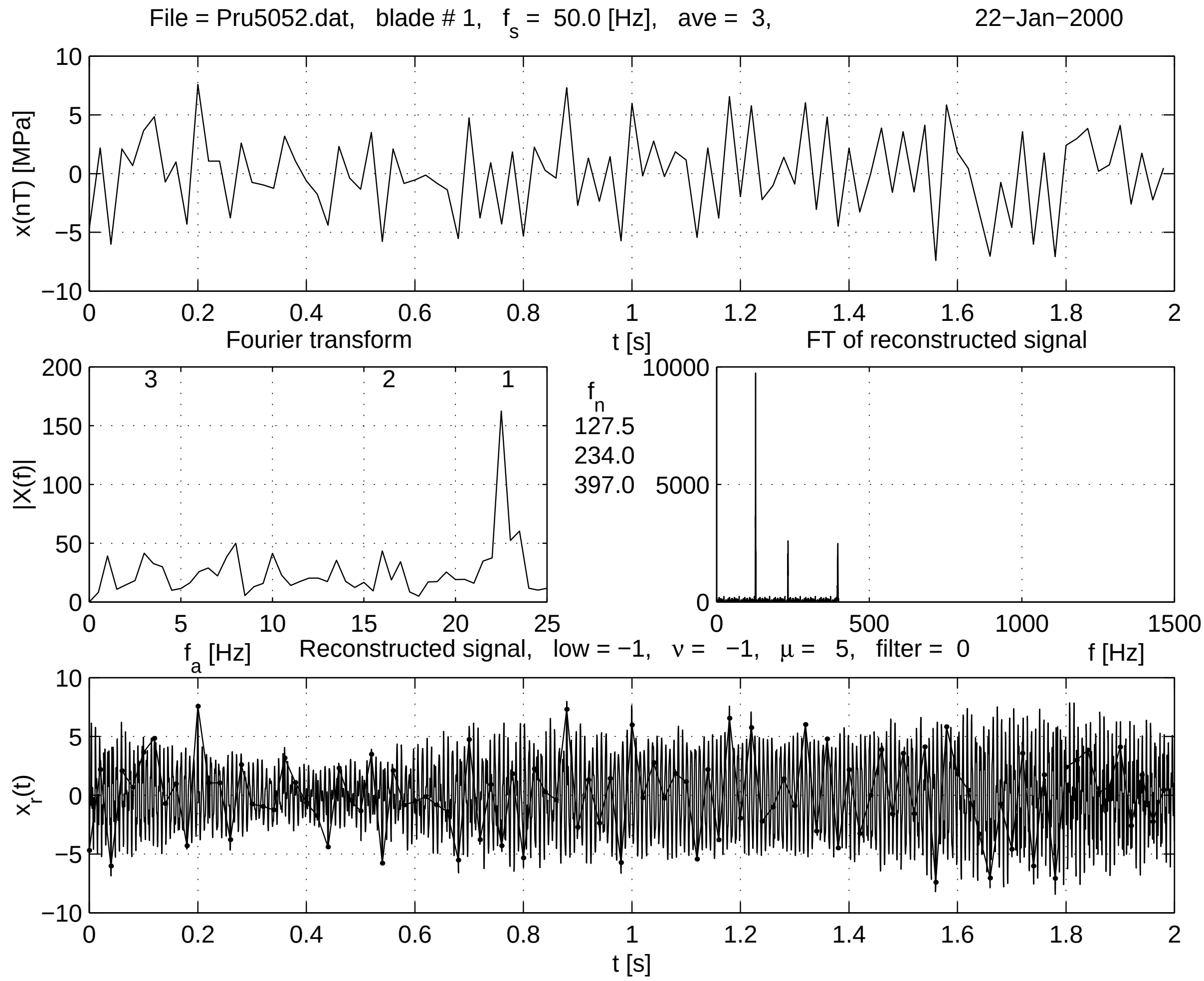

Figure 4: Slowly sampled signal, its FT, reconstructed FT, and reconstructed signal of a real blade

4 DAMAGE ESTIMATION

An estimation of a damage of blades due to vibration is a delicate complex problem. There are the following reasons for such a statement:

a. Blade tip deflections are composed out of all frequency components of the spectrum.
b. Every mode of vibration has its own critical places, where extreme stress occurs. In general, they are mutually different,
c. The individual modes need not generate uniaxial stress in the critical points.
d. Even in the case of one dominant component of principal stress, the damage may be estimated by many different formulae giving different resulting damage.
e. The stress varies at random in time and owing to conditions of operation.
f. Material properties are known with a large variance.

Let us suppose the stress in critical places of a blade are almost uniaxial. A damage of a blade material is dependent on the area of hysteretic loops in the stress-deformation plot. The area is proportional to an energy spent for plastic deformations, which are a measure of the damage.

The Figure 5 shows how important is the knowledge of hysteresis loops of a complex signal. Heights of the loops may be found by a decomposition of the random stress signal into the full cycles by a procedure, which has been named by its authors a "rain flow" method (5). In the middle of seventieth, the author of this contribution has found the way, how to decompose the random process into full cycles in real time of measurements. The standard method has been singe-channel oriented.

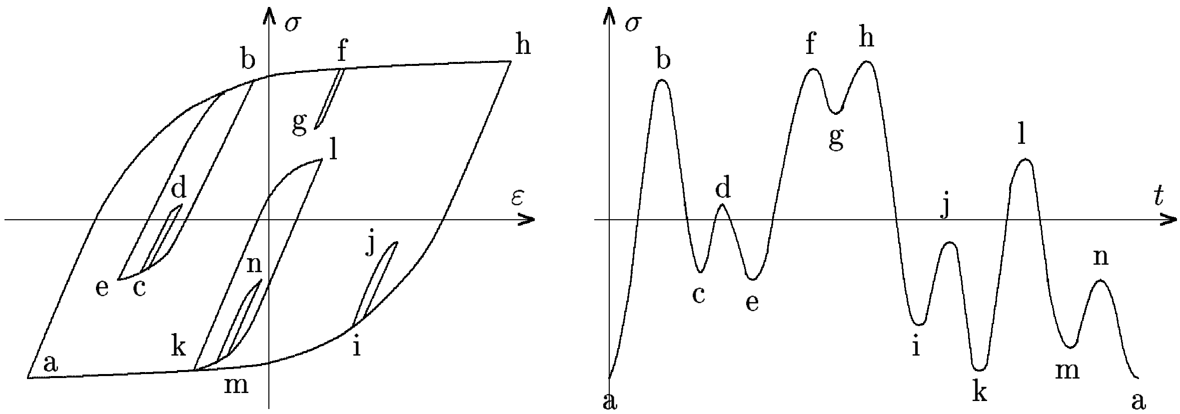

Figure 5: Random stress and corresponding hysteretic loops

The general single-channel rain flow method is based on the application of a special memory called the stack, where incoming local extremes of a random process are stored until the extreme on the top of the stack (TOS) creates the full cycle with an extreme just below TOS. That pair of extremes creating a full cycle is then removed from the stack and counted on the proper place of 2D histogram, a matrix of full cycle counts. Then, the TOS is tested again for an existence of a full cycle till there is any. After all, the new extreme is put on TOS, and the whole procedure is repeated. A long-term using of the rain flow method for random stress decompositions has revealed cases, in which a multi-channel rain flow method would be necessary in practice. An optimization of structures exposed to random loading, and an estimation of a damage of vibrating blades belong to such tasks. This has been the reason why the multi-channel rain flow method has been developed.

The new method changes a bit the processing of full cycles, and calculates a relative damage of every blade instead of counting histograms, because the monitoring system should give information on the damage level in any time. For the purpose, a partial relative damage is evaluated as soon as the full cycle occurs. In spite of an existence of plenty of hypothesis for an estimation of the damage caused by a single cycle, the Palmgren-Miner law of linear damage cumulation is the most simple and popular. Therefore, the relative damage of the *i*-th full cycle of the amplitude s_{ai} and mean value s_{mi} is evaluated using the formula

$$d_i = \frac{1}{N_{cmi}} \left(\frac{s_{ai}}{s_{cmi}} \right)^{w_{mi}}, \qquad (10)$$

where s_{cm} is a fatigue limit, N_{cm} a number of harmonic cycles up to the breakdown with amplitudes $s_a = s_{cm}$, and w_m an exponent of the S-N curve. The subscript *m* denotes that those quantities depend on the mean stress s_m. The cumulative relative damage of the *j*-th blade is estimated as a sum of all relative damages in the given critical place as

$$D_j = \sum_i d_{ij} \,, \tag{11}$$

where D_j should not exceed a constant k_d determined by a machine producer. Should the Palmgren-Miner law held perfectly, k_d would approach unity. The residual time of life of a bladed disc is determined through the blade showing the shortest life or the greatest relative damage.

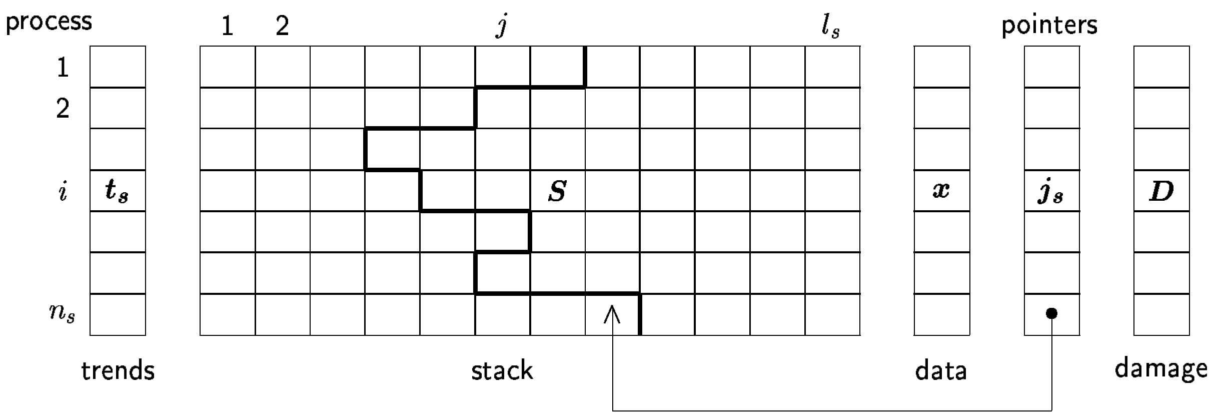

Figure 6: Memory requirements for a decomposition of n_s signals

The main idea of the multi-channel rain flow method compared with the single-channel one remained unchanged. Owing to this fact, there should be as many parallel stacks as a number of observed objects (blades) n_s. They constitute a stack matrix $\boldsymbol{S} \in R^{n_s, l_s}$, the rows of which are stacks belonging to individual blades. The number of columns l_s of the matrix $\boldsymbol{S}$ should be sufficient for storing extremes, which have not built full cycles yet. Next to it, several vectors, all of the length n_s, are necessary. Since the full cycles may occur on the blades in different times, a vector $\boldsymbol{j}_s$ of pointers to tops of the stacks should exist. A vector $\boldsymbol{t}_s$ contains zeros or ones expressing whether the particular signal just tends to minimum or maximum respectively. $\boldsymbol{D}$ might be vector or matrix containing the cumulative relative damages D_j in critical points. There is still one vector in the Figure 6, vector $\boldsymbol{x}$, which holds recent samples of all signals, and serves for a recognition full cycles on tops of stacks before being stored there.

5 CONCLUSIONS

The paper describes a new method for evaluating a relative damage accumulated in critical points of all blades of one rotating turbine disc out of sparsely sampled positions of their tips provided by a blade monitoring system. The measured signals are reconstructed with the help of information on approximate resonant frequencies by reconstructing aliased Fourier spectra.

The reconstructed signals contain as many resonant frequencies as have been applied for. The relative damage is estimated from the Palmgren-Miner law of linear cumulation of elementary damages caused by closed hysteretic loops found by means of a multi-channel rain flow method applied to the reconstructed signals.

The problems of a monitoring system itself, sensors, strategy of data acquisition and a consequent decisions in a control room of a power plant would need another space and time to be dealt with.

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