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Methods for determination of Lamb wave dispersion curves by means of Fourier transform

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The application of the conventional ultrasonic methods, such as pulse-echo, has been limited to testing relatively simple geometries or interrogating the region in the immediate vicinity of the transducer. A new ultrasonic methodology uses guided waves to examine structural components. The advantages of this technique include: its ability to test the entire structure in a single measurement; and its capability to test inaccessible regions of complex components.

The propagation of guided waves in a complex structure is a complicated process that is difficult to understand and interpret. The current research develops the mechanics fundamentals that model this propagation.

One approach to modeling guided wave propagation phenomena is to analytically solve the governing differential equations of motion and their associated boundary conditions. This procedure already has been done for simple geometries and perfect specimens without defects (see [2] and [3]). However, these equations become intractable for more complicated geometries or for a non-perfect specimen.

Another approach to this problem is a numerical solution. A number of different numerical computational techniques can be used for the analysis of wave propagation. These include finite difference equations, finite element methods (FEM), finite strip elements, boundary element methods, global matrix approaches, spectral element methods, mass-spring lattice models and the local interaction simulation approach. The primary advantage of the FEM is that there are numerous available commercial FEM codes, thus eliminating any need to develop actual code.

Main feature of the guided wave method is comparison of dispersion curves of a perfect structure with a defect structure. This contribution reports on methods for determination of Lamb wave dispersion curves by means of Fourier transform (FT).

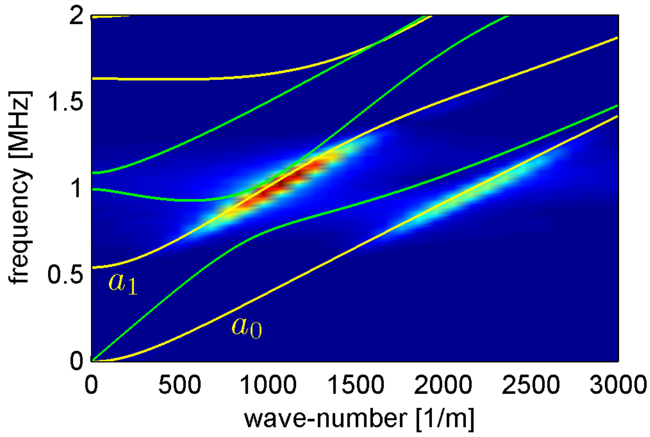
Lamb waves are two–dimensional propagating vibrations in free plates, with displacements that may be symmetric or antisymmetric with respect to the middle of the plate, and are eigen-solutions of characteristic equations, hence the term free or normal modes. The velocities of all Lamb waves are dispersive.

Propagating Lamb waves are sinusoidal in both the frequency domain and the spatial domains. Therefore, a temporal FT may be carried out to go from the time to the frequency domain, and then a spatial FT may be carried out to go to the *frequency–wave-number* domain, where the amplitudes and the wave-numbers of individual modes may be measured.

The discrete 2D FT may be defined in a similar to the 1D FT. The result of this transformation will be a 2D array of amplitudes at discrete frequencies and wave-numbers.

The algorithm: 1) Create the array (in column order) from gained the time histories of the waves received at a series of equally spaced positions along the propagation path. 2) Carry out a temporal FT of each column to obtain a frequency spectrum for each position. At the stage, an array with the spectral information for each position in its respective column is obtained. 3) Carry out a spatial FT of each row formed by the components at a given frequency to obtain the *frequency–wave-number* information.

For demonstration of this algorithm, a relatively simple geometry is considered: the 3 mm thick and 125 mm long steel plate. This geometry has the advantage of a known analytical solution (the Rayleigh-Lamb equation). The FEM program used for this work was COMSOL Multiphysics [1]. The left edge of this plate, which is modeled with quadrilateral elements (0.5×0.5 mm), is loaded by displacements equivalent to the second antisymmetric mode (a_1). The time function of these displacements is a 5 cycle, 1 MHz sinusoidal tone burst modulated temporally by a Hanning window function. This FEM task is solved in time domain. The obtained vertical displacements at nodes on the upper surface of the plate are considered for the 2D FFT algorithm (created by MATLAB [4]).



The figure shows the pseudocolor plot of *frequency–wave-number* spectrum. The dispersion curves obtained analytically are plotted by solid lines (symmetric mode – green, antisymmetric – yellow). Note that two antisymmetric modes (a_1 and a_0) exist at given frequency (1 MHz).

Other method of determination of dispersion curves is based on solution of FEM task in frequency domain. The frequency spectrum of a time–dependent excitation is defined using FT. Then, the structure response (complex displacements and stresses at any locations) is calculated at each frequency of this load, i.e. the FE–code supplies stationary solution for each frequency component of the temporal excitation. A specific signal processing is then applied to these FE data to recover the wave-numbers of the various guided modes propagating along the plate. Algorithm: 1) Create the array (in column order) from gained the complex displacement for each frequency received at a series of equally spaced positions along the propagation path. 2) Carry out a spatial FFT of each row to obtain the *frequency–wave-number* diagram.

Since a small number of frequencies are sufficient to achieve a correct representation of a wide variety of temporal excitations, this approach considerably speeds up the computation by avoiding the temporal FFT and by decreasing the number of calculation steps.

Acknowledgements

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References

[1] COMSOL, Inc., http://www.comsol.com

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[4] The MathWorks, Inc., http://www.mathworks.com