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Comparison of three analytical methods based on three basic tasks of elastodynamics

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The main aim of the contribution is to illustrate three various methods of solutions on three tasks of the nonstationary state of stress. The advantages and drawbacks of these methods will be shown. The following tasks will be investigated: a) uniform string, plucked aside at its centre; b) torsionally loaded disc with concentric hole; c) point loaded thick plate.

Problem of a uniform string, plucked aside at its centre: This example is taken from problems given in Rayleigh's *The Theory of Sound*. The Laplace transform of transverse vibrations u is given

$$\bar{u}(p,x) = \frac{kx}{p} - \frac{kc}{p^2}\frac{\sinh\frac{px}{c}}{\cosh\frac{pl}{2c}}, \tag{1}$$

where c is velocity of transverse vibrations, l is length of string, p is parameter of Laplace transform and $k = 2h/l$, where h is initial displacement of string.

When the Laplace transform of solution has been determined, the expression of u by means of a sum of residues will lead to the expansion of u in a series of normal functions.

$$u(t,x) = \frac{4kl}{\pi^2}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^2}\sin\left\{(2n+1)\frac{\pi x}{l}\right\}\cos\left\{(2n+1)\frac{\pi tc}{l}\right\}, \tag{2}$$

Bromwich [3] found alternative method. It is often possible to replace the expression for $\bar{u}$ by a series of terms such as e^{-ap}/p^{n+1}, where a is real and n is an integer. Then

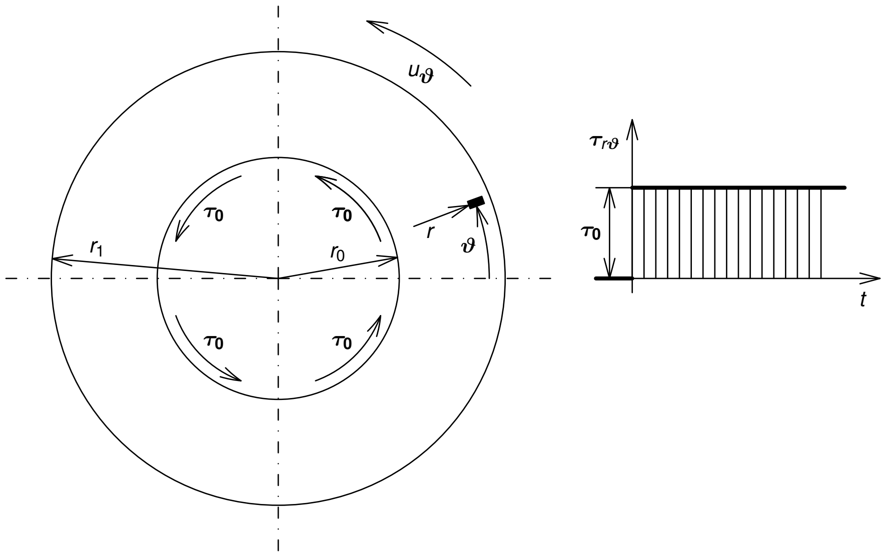
$$\bar{u}(p,x) = \frac{kx}{p} - \frac{kc}{p^2}\left\{e^{-p/c(l/2-x)} - e^{-p/c(l/2+x)}\right\}\left(1 - e^{-pl/c} + e^{-2pl/c} - \ldots\right), \tag{3}$$

Thus we find

$$u(t,x) = \begin{cases} kx & (0 < t < \frac{1}{2}l - x), \\ k(\frac{1}{2}l - ct) & (\frac{1}{2}l - x < t < \frac{1}{2}l + x), \\ -kx & (\frac{1}{2}l + x < t < \frac{3}{2}l - x), \\ k(ct - \frac{3}{2}l) & (\frac{3}{2}l - x < t < \frac{3}{2}l + x), \\ kx & (\frac{3}{2}l + x < t < \frac{5}{2}l - x), \\ \text{and so on.} \end{cases} \tag{4}$$

The numerical inverse Laplace transform (NILT) applied to eq.(1) is third method as to solve this task. The FFT based NILT with ϵ-algorithm algorithm for accelerating convergence of the residual infinite series was used for this purpose [1].

Problem of a torsionally loaded disc with concentric hole: This task is solved in [2] by means of a sum of residues. In contribution will be given both these results and the results from the numerical inverse Laplace transform. Among others, NILT has advantage while evaluating velocities or accelerations because the time derivations are not needed.



Problem of a point loaded thick plate: This task will also be solved by means of a sum residues. The numerical work involved in this analysis is long and difficult. The method is more effective for long-time transient responses at remote observation points. In addition, a generalized ray theory [5] will be used. This method is based on the *Bromwich expansion method* [3]. The ray integrals for transient waves (given from *Bromwich expansion*) are evaluated by applying the so-called *Cagniard's method* [4]. The solution is exact up to the time of arrival of the next ray.

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