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Robust method for finding of dispersion curves in a thick plate problem.

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Waves propagation in thick plates is well solvable problem[2]. One of part of task solution is to find the waveform dispersion curves. This problem has been chosen for the first trial using interval arithmetic[3], because of its relative simplicity. In calculating the dispersion curves is needed to quantify the only trigonometric functions, hyperbolic functions, and square roots. All of these functions are already included in INTLAB[4] and therefore need not to be newly programmed.

The thick plate is defined as that it has a nonzero thickness d and endless remaining dimensions. To calculate the stress wave propagation in plates it is used the integration along the dispersion curves for thick plates. This dispersion relations is defined as

$$\left(\xi^2-2\right)^2\tanh\left(kd\sqrt{1-\xi^2}\right)-4\cdot\sqrt{1-\xi^2}\sqrt{1-\kappa\xi^2}\tanh\left(kd\sqrt{1-\kappa\xi^2}\right)=0,$$

where κ mean the ratio of the squares of the phase velocities for the plate's material. Due avoid computation in the complex domain, because of implementation in INTLAB, it was necessary to divide the dispersive depending equations according to the value of ξ. For each interval are the equations shown in tab 1. To quantify the equations and finding the waveform dispersion curves is used interval arithmetic.

Tab. 1. Dispersion equation
$\left(\xi^2-2\right)^2\cdot\alpha-4\cdot\beta\cdot x_1x_2=0$

	$0<\xi<1$	$1<\xi<\frac{1}{\sqrt{\kappa}}$	$\frac{1}{\sqrt{\kappa}}<\xi$
x_1	$\sqrt{1-\xi^2}$	$\sqrt{\xi^2-1}$	$\sqrt{\xi^2-1}$
x_2	$\sqrt{1-\kappa\xi^2}$	$\sqrt{1-\kappa\xi^2}$	$\sqrt{\kappa\xi^2-1}$
α	$\sinh(kdx_1)\cosh(kdx_2)$	$\sin(kdx_1)\cosh(kdx_2)$	$\sin(kdx_1)\cos(kdx_2)$
β	$\cosh(kdx_1)\sinh(kdx_2)$	$\cos(kdx_1)\sinh(kdx_2)$	$-\cos(kdx_1)\sin(kdx_2)$

Interval arithmetic is an extension of arithmetic over real numbers, where for each real function $f(x_1,\ldots,x_n)$, the interval function $F(X_1,\ldots,X_n)$ is called an interval extension of the function f if for each intervals $I_1,\ldots,I_n$ function $F(I_1,\ldots,I_n)$ returns interval I such that $\forall y_1\in I_1\ldots\forall y_n\in I_n(f(y_1,\ldots,y_n)\in I)$. For other applications it is particularly important the interval Newton method, according to which each continuously differentiable function f

each interval I must be $\forall a, x \in I, \exists \epsilon \in I(f(x) = f(a) + (x-a)f'(\epsilon)$. Specifically then, the function f continuously differentiable on the interval I has all the roots in I in interval $N_a = a - (F(a)/F'(I))$ where a is an arbitrary element of I and F, F' is an interval extension function of f, f'.

For quantification of interval arithmetic was used MATLAB's toolbox INTLAB, which are defined not only the basic functions for interval arithmetic, but their automatic differentiation too [1].

When trying to use the Newton method in INTLAB was needed to solve the problem by dividing intervals. INTLAB always returns the result $\langle -\infty, \infty \rangle$ for each I/J where $0 \in J$. However, for this case, it was necessary defined alternative way of dividing, when the interval is divided into two portions, and finding roots thus diverges into two tasks. The actual calculation is solved in recursive steps. In a single step in the equation $N_a = a-(F(a)/F'(I))$ is substituted middle I per a, yielding a new interval $J = I \cap N_a$. This one is used in the next recursive step. The calculation continues until the result interval width falls under a predetermined accuracy.

Interval arithmetic provides a robust method for finding the roots of the dispersion equation. For relatively low speed, this method is not suitable to complete the calculation, it is however possible to use the first approach with low accuracy, or when using the Gaussian integration method.

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