A LOCAL TIME STEPPING FOR DISCONTINUOUS WAVE PROPAGATION IN A HETEROGENEOUS BAR

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At this time, additive technology is acceptable as a technology for manufacturing of complex bodies with complicated shapes and various and advanced properties as smart materials, where conventional technologies are not possible to use for manufacturing. One could find applications of 3D printed bodies in mechanical, biomechanical or aerospace engineering and in many others. For that reason, understanding to wave processes in heterogeneous, layered and functionally graded materials is important issue [1]. Based on this knowledge, heterogeneous bodies under dynamic and shock loading can be designed to hold optimal properties for real applications with respect to utilization, termo-mechanical behaviours, lifetime, cost and so on.

Wave propagation problems in graded elastic bars has been studied in [2]. The accurate modelling of discontinuous wave propagation in elastic heterogeneous bodies is still an open problem in numerical methods. In the context of the numerical modelling, complications arise from spurious stress oscillations, different wave speed in each material point, varying local stability conditions with obvious consequences on the discretization scheme, and dispersion errors. The numerical methods currently used comprise the finite volume method [3], spectral methods, higher order discontinuous Galerkin formulation, the graded finite element method [4] and many others. In this paper, we adopted the Park's scheme with pullback time integration presented in [5]. In this method, the local stepping algorithm with respect to local wave speed and local stability condition at each material point is employed [6].

1 Wave propagation in graded bar with linear distribution of elastic modulus

We study discontinuous wave propagation in a graded bar with the linear distribution of elastic modulus and constant mass density. The analytical solution of the problem can be found in [7]. The length of the bar is $L = 1$ m. In this test, the mass density is chosen as $\rho = 1$ kg/m^3. The elastic modulus on the left side of the bar is $E_1 = 1$ Pa and on the right side it is set as $E_2 = 0.25$ Pa. The bar is loaded on the left side by the Heaviside pulse with the stress amplitude as $\sigma_0 = 1$ Pa. Results of numerical solution of the elastic wave propagation problem in the graded elastic bar are presented in Fig. 1 for time $T = 0.75$ s obtained by the analytical solution [7], semi-analytical solution with the numerical inverse Laplace transformation [8], the finite volume method [3], the finite element method with explicit time integration by the central difference method, the finite element method with the Park's method with and without the local time stepping [6]. For FEM and FVM, the time step size was set as a minimum value of local stable time steps over all elements/cells.

In the numerical test, the presented explicit scheme with local time stepping produces results with improving spurious oscillations in the finite element method. We observed only cusps on wavefront of stress discontinuities. Further, the improvement of stress spurious oscillations is evident with comparison of the scheme with and without local time stepping, because the local stepping time process respects local critical time step size at each material point.

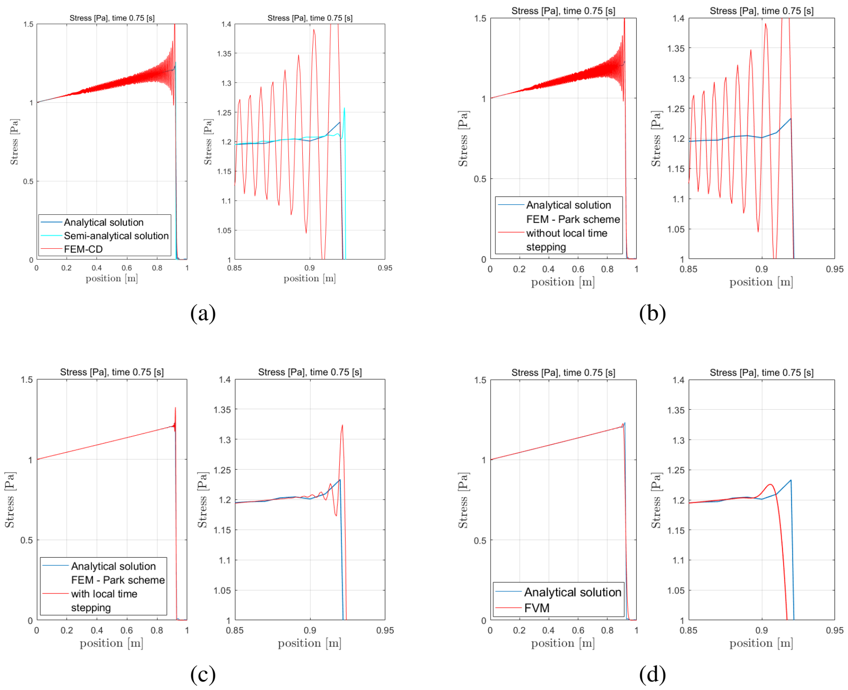

Figure 1: Stress distributions in a graded bar obtained by a) FEM with the central difference method (CD), b) FEM with the Park method without local time stepping, c) FEM with the Park method with local time stepping, and d) FVM.

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